Citation:
C. A. Cruz-Villar, J. Álvarez-Gallegos, M. G. Villarreal-Cervantes, "Concurrent redesign of an underactuated robot manipulator", Mechatronics, Vol. 19, No. 2, pp. 178-183, ISSN: 0957-4158, DOI:10.1016/j.mechatronics.2008.09.002 , March 2009.

Concurrent redesign of an underactuated robot manipulator

Carlos A. Cruz-Villar [a], Jaime Alvarez-Gallegos [a], Miguel G. Villarreal-Cervantes [a,*]

[a] *Mechatronics Section, Electrical Engineering Department, Cinvestav-IPN, Av. Instituto Politécnico Nacional, No. 2508, Col. San Pedro Zacatenco, México D.F. 07360, Apdo. Postal 14-740*

Abstract

An interesting problem in robotics is to minimize the required time to force a manipulator to travel between two specific points (positioning time). In this paper a concurrent structure-control redesign approach is proposed in order to find the minimum positioning time of an underactuated robot manipulator, by considering a synergetic combination between the structural parameters and a bang-bang control law. The problem consists in finding the structural parameters of the system and the switching intervals of the bang-bang control that simultaneously minimize the positioning time, subject to the input constraint and the structural parameter constraint. The concurrent structure-control redesign approach is stated as a dynamic optimization problem (DOP). The projected gradient method is used to solve the DOP.

The effectiveness of the proposed concurrent redesign approach is shown via simulation and experimental results on an underactuated system called the Pendubot.

Key words: Concurrent Redesign; Design Optimization; Minimum Time; Pendubot

1 Introduction

Robots are currently used for a variety of tasks in industrial applications. Improving their performance could result in commercial benefits.

* Corresponding author. Tel.:+52 5550613800 ext. 6311.
Email address: gvillarr@cinvestav.mx (Miguel G. Villarreal-Cervantes).
URL: http://www.meca.cinvestav.mx/ (Miguel G. Villarreal-Cervantes).

Time-optimal control of robots [1,2] has been used in order to minimize the required time to take the system from an initial state to a final one (positioning time). To find the time-optimal control, infinite dimensional problems are usually stated. A variational approach, which focuses on obtaining a solution via the classical necessary conditions for optimality, solves the infinite dimensional problem. It is proven in [4] that the behavior of the control signal for time-optimal control problems with bounded control input and with terminal constraints is of bang-bang nature [1]. Then, time-optimal control problems can be parametrized [3] in order to transform them into finite dimensional problems. So these finite dimensional problems can be solved by using nonlinear programming (NLP) methods.

On the other hand, in the last years, an integrated design approach [5–8] which involves a structure-control design, has been employed to improve the performance of electromechanical systems. This approach consists in integrating the design parameters of both the mechanical structure and the controller into a single design process.

Fu and Mills [5] proposed a structure-control design approach based on convex optimization theory, a linear model of the system and closed-loop control laws. In that approach, the design specifications (performance index vector) must be convex with respect to the closed-loop transfer function. Rastegar et al. [6] optimized the kinematic, dynamic and closed-loop control parameters with an integrated design approach in order to obtain a high performance robotic system that rapidly executes point-to-point motions, with minimal vibration. Ravichandran et al. [7] presented a methodology based on numerical optimization techniques for simultaneously optimizing design parameters of a two-link planar rigid manipulator and nonlinear PD controllers. Both design parameters are optimized to perform multiple tasks. In Alvarez et al. [8] a structure-control integrated approach is used in order to design a pinion-rack continuously variable transmission. A multi objective dynamic optimization problem is stated in order to concurrently obtain a set of optimal mechanical structure and control parameters.

The previous works share a common characteristic: the parameters of a closed-loop control law were taken into account in the integrated design approach. In this paper, a concurrent redesign approach is proposed. This approach involves a parametric redesign procedure of the system considering structural and controller parameters. Hence the open-loop bang-bang control parameters and the structural parameters are simultaneously found in this approach. Therefore, the concurrent redesign approach is formulated as a

[1] A control signal which is restricted to be in a lower and an upper bound and it switches immediately from one extreme to another at certain times (the control signal is never strictly between the bounds), is called bang-bang control

dynamic optimization problem, where the design variables are both the structural parameters of the system and the switching time interval vector of the bang bang control law and the redesign objective is to minimize the positioning time of a nonlinear electromechanical system, subject to the system dynamics. The numerical projected gradient optimization method is used to solve such a dynamic optimization problem. To apply this approach, it is assumed that the trajectory from the initial state to the final one must be obstacle free, that is, without trajectory restrictions and that dynamic system model is a smooth and differentiable one.

Underactuated systems arise in many circumstances such as when the system is underactuated by design, or when the actuators are expensive and/or heavy, such that they are avoided in the system design, or due to an actuator failure. If an actuator fails, a solution via hardware or software could be implemented. A solution via hardware may be performed by equipping the system with redundant actuators, adding in consequence weight to the system. On the other hand, a solution via software is a cost and weight-reducing alternative, since only a switching of the control law is required when an actuator failure is detected. So, control laws for underactuated manipulators could provide fault tolerance to the system against actuator failures [9].

In this paper, it is proposed a concurrent redesign approach to "the swinging up" problem of an underactuated double pendulum system called Pendubot.

The remainder of this paper is organized as follows, the concurrent redesign approach is presented in section 2. Dynamic equations of the Pendubot and link parameters (masses, mass center lengths, inertias) with the structural modifications are found in section 3. Simulation and experimental results are found in section 4. Conclusions of this paper are presented in section 5.

2 Concurrent redesign approach for system positioning in minimum time

The concurrent redesign approach consists in finding the input control signal $u(t) \in R$ for all time $t_0 \leq t \leq t_f$ and the structural parameters $\Upsilon \in R^{n_\Upsilon}$ of an electromechanical system, that bring the system from the initial conditions $x(t_0) = x_0$ to the final one $x(t_f) = x_f$ in minimum time. In this paper, the redesign problem is formulated as a dynamic optimization problem (DOP) consisting of minimizing under $u(t)$ and Υ, the functional (1) subject to the nonlinear dynamic equation of the electromechanical system (2), the initial state (3), the final state (4), the input control signal constraint (5) and the structural parameter constraints (6), where $x(t) \in R^n$ is the state vector, $u(t)$ is the input control signal, Υ is the structural parameter vector, x_0 and x_f are the initial and the final desired states respectively.

$$J = t_f \tag{1}$$
$$\dot{x} = f(x(t), u(t), \Upsilon) \tag{2}$$
$$x(t_0) = x_0 \tag{3}$$
$$x(t_f) = x_f \tag{4}$$
$$u_{\min} \leq u(t) \leq u_{\max} \tag{5}$$
$$\Upsilon_{\min} \leq \Upsilon \leq \Upsilon_{\max} \tag{6}$$

To transform the constrained problem stated above into an unconstrained one, barrier functions [10] are considered. These barrier functions set a barrier against the solution leaves the feasible region imposed by the constraints when they are added to the performance index of the optimization problem. Therefore, the dynamic optimization problem stated in (1)-(6) is reformulated in (8)-(11), which considers the inequality constraints (6) as barrier functions. Moreover, defining the input control vector $u(t)$ as a bang-bang control law, the switching time interval vector Δt of the control signal must be included in the next problem reformulation. Then $u(t)$ is stated in the following way:

$$u(t) = \begin{cases} u_{\max} \text{ if } \Delta t_j > 0 \\ u_{\min} \text{ if } \Delta t_j < 0 \end{cases} \tag{7}$$

The sign of the switching interval Δt_j, obtained by the optimization algorithm (12), could have a positive or negative sign representing the corresponding control signal ($u_{\min}$ or $u_{\max}$), and the absolute value of the switching interval $|\Delta t_j|$ represents the time interval length when the corresponding control signal ($u_{\min}$ or $u_{\max}$) is applied.

Then, the problem reformulation of the dynamic optimization problem (1)-(6) consists in finding the switching time intervals of the control signal $\Delta t = [\Delta t_1, ..., \Delta t_{n_{\Delta t}}]^T \in R^{n_{\Delta t}}$ and the structural parameters $\Upsilon = [\Upsilon_1, ..., \Upsilon_{n_\Upsilon}]^T \in R^{n_\Upsilon}$ that minimize the modified performance index (8) subject to the nonlinear dynamic equation of the electromechanical system (9), the initial conditions (10) and the final state $x(t_f)$ constraints (11), with $u(t)$ as defined in (7) and $t_f \equiv \sum_{j=1}^{n_{\Delta t}} |\Delta t_j|$. In equation (8), $\bar{\Xi} = \sum_{l=1}^{n_\Upsilon} \underset{l}{\Xi}$ is the barrier function set for the structural parameter vector Υ, where $\underset{l}{\Xi} = \omega_{\Upsilon_l}\left[\frac{1}{-(\Upsilon_l - \Upsilon_{l\max})} + \frac{1}{-(-\Upsilon_l + \Upsilon_{l\min})}\right]$ is the barrier function for the $l-th$ structural parameter, $\Upsilon_{\min} = [\Upsilon_{1_{\min}}, \ldots, \Upsilon_{n_{\Upsilon_{\min}}}]^T \in R^{n_\Upsilon}$ and $\Upsilon_{\max} = [\Upsilon_{1_{\max}}, \ldots, \Upsilon_{n_{\Upsilon_{\max}}}]^T \in R^{n_\Upsilon}$ are the minimum and maximum limit vectors for the structural parameter Υ and $\omega_\Upsilon = [\omega_{\Upsilon_1}, \ldots, \omega_{\Upsilon_{n_\Upsilon}}] \in R^{n_\Upsilon}$ is a constant vector with elements as close to zero as possible, according to the barrier function approach to deal with inequality constraints [10].

$$J(\Delta t,\Upsilon) = t_f + \bar{\Xi} = \sum_{j=1}^{n_{\Delta t}} |\Delta t_j| + \sum_{l=1}^{n_\Upsilon} \underset{l}{\Xi} \tag{8}$$

$$\dot{x} = f(x(t),\Delta t,\Upsilon) \tag{9}$$

$$x(t_0) = x_0 \tag{10}$$

$$e_f = x(t_f) - x_f = 0 \tag{11}$$

As equation (8) and (9) depend explicitly on the parameters Δt and Υ, changes in those parameters would modify the system performance.

In this work, the projected gradient algorithm (12) is used to solve the problem stated above, where $k_r, k_n \in R^{(n_{\Delta t}+n_\Upsilon)\times(n_{\Delta t}+n_\Upsilon)}$ are diagonal matrices with the step sizes $k_{r_{ii}}$, $k_{n_{ii}}$ for $i = 1,...,n_{\Delta t}+n_\Upsilon$ in the diagonal elements. I is the identity matrix, $\breve{A} = \left[\frac{\partial x(t_f)}{\partial \Delta t}, \frac{\partial x(t_f)}{\partial \Upsilon}\right] \in R^{(n)\times(n_{\Delta t}+n_\Upsilon)}$ is the Jacobian matrix of partial derivatives of $x(t_f)$ with respect to Δt and Υ, $\nabla J = \left[\frac{\partial t_f}{\partial \Delta t}, \frac{\partial \bar{\Xi}}{\partial \Upsilon}\right]^T$ is the gradient of J with respect to Δt and Υ, $P = \left(I - \breve{A}^+\breve{A}\right)$ is the projection matrix of the gradient of the performance index onto the null space of the constraint matrix $\breve{A}$, $\breve{A}^+$ is the Moore-Penrose pseudo-inverse of $\breve{A}$ ($\breve{A}^+ = \breve{A}^T(\breve{A}\breve{A}^T)^{-1}$ if $n \leq n_{\Delta t}+n_\Upsilon$, or $\breve{A}^+ = (\breve{A}^T\breve{A})^{-1}\breve{A}^T$ if $n > n_{\Delta t}+n_\Upsilon$), and $\breve{A}^+e_f$ is the solution that minimizes the error $e_f = x(t_f) - x_f$. The convergence of (12) to a solution is guaranteed with small step sizes k_r and k_n [10], that is, the constraint error e_f and the projection $P\Delta J$ converge to zero. Then, the step sizes $k_{r_{ii}}$, $k_{n_{ii}}$ are selected to be in the interval $(0,1]$. In order to obtain the optimal parameters with less computing time, the constraint error fulfillment e_f must have more importance than the projection $P\Delta J$ in (12), that is, $k_{r_{ii}} \geq k_{n_{ii}}$ must be taken into account.

$$\begin{bmatrix} \Delta t_{new} \\ \Upsilon_{new} \end{bmatrix} = \begin{bmatrix} \Delta t \\ \Upsilon \end{bmatrix} - k_r\breve{A}^+e_f - k_nP\nabla J \tag{12}$$

The sensitivity of $x(t_f)$ with respect to each component Δt_j (13) of the switching interval vector Δt can be obtained as in [1], where $sgn(.)$ is the sign function, $x_j \equiv x(t_j)$ and $\dot{x}_j = f(x_j(t),\Delta t_j,\Upsilon)$. $A_j(t_j) = \frac{\partial x_j}{\partial x_{j-1}}$, $\frac{\partial x(t_f)}{\partial x_j} = A_{n_{\Delta t}}A_{n_{\Delta t}-1}...A_{j+1}$ by the chain rule.

$$\frac{\partial x(t_f)}{\partial \Delta t_j} = \frac{\partial x(t_f)}{\partial t_j}\frac{\partial t_j}{\partial \Delta t_j}\frac{\partial x_j}{\partial x_j} = \frac{\partial x(t_f)}{\partial x_j}\dot{x}_j sgn(\Delta t_j) \tag{13}$$

Hence, the matrices A_j can be computed by integrating the first variation of (9). If $A_j(t) = \frac{\partial x(t)}{\partial x_{j-1}}$, $t_{j-1} \leq t \leq t_j$, then A_j satisfies (14), with the initial condition $A_j(t_{j-1}) = I$ for each $t_{j-1} \leq t \leq t_j$, where I is the identity matrix, $A_j = A_j(t_j)$ and $x(t)$ is the trajectory resulting from Δt.

$$\frac{dA_j(t)}{\partial t} = \left[\frac{\partial f(x(t), \Delta t_j, \Upsilon)}{\partial x}\right] A_j(t) \tag{14}$$

The sensitivity of $x(t_f)$ with respect to each structural parameter Υ_l can be obtained from (15), with the initial condition $\frac{\partial x(t_0)}{\partial \Upsilon_l} = 0$.

$$\frac{d\left(\frac{\partial x(t)}{\partial \Upsilon_l}\right)}{dt} = \frac{\partial f(x(t), \Delta t, \Upsilon)}{\partial x(t)} \left(\frac{\partial x(t)}{\partial \Upsilon_l}\right) + \frac{\partial f(x(t), \Delta t, \Upsilon)}{\partial \Upsilon_l} \tag{15}$$

In this work the mechanism proposed in [1] is used to add or delete switching intervals in the projected gradient algorithm (12). As the concurrent redesign optimization problem is a nonlinear one, the projected gradient algorithm (12) with different starting conditions (Δt_{ini}, Υ_{ini}) could converge to different local minima ($\Delta t^{\star}$, $\Upsilon^{\star}$). Therefore, a procedure to select appropriate starting conditions should be used.

3 Dynamic system and structural parameters

The Pendubot (Fig. 1) is an underactuated double pendulum system with one actuator at the first joint and no actuator at the second one. The lack of an actuator severely modifies the dynamic behavior of the system which, may become chaotic [11].

The Pendubot proposal was in 1991 [12]. Since that time, one control objective has been to bring the links from a given initial configuration, namely the "downwards" position or rest position, to an arbitrary final position, in particular the "upwards" position. The swinging up control consists in bringing the links from the downwards position ($q_1 = -\pi/2$ $[rad], q_2 = [rad]$, in Fig. 1) to the upwards position ($q_1 = \pi/2$ $[rad]$, $q_2 = 0$ $[rad]$, in Fig. 1). Another control law, which is called the balance control, is then required in order to stabilize the links in the upwards position. In this paper the swinging up control is the bang-bang control obtained via the concurrent redesign approach and the balance control is a linear quadratic regulator (LQR). Block [12] proposed a strategy for switching from the swinging up control to the balance

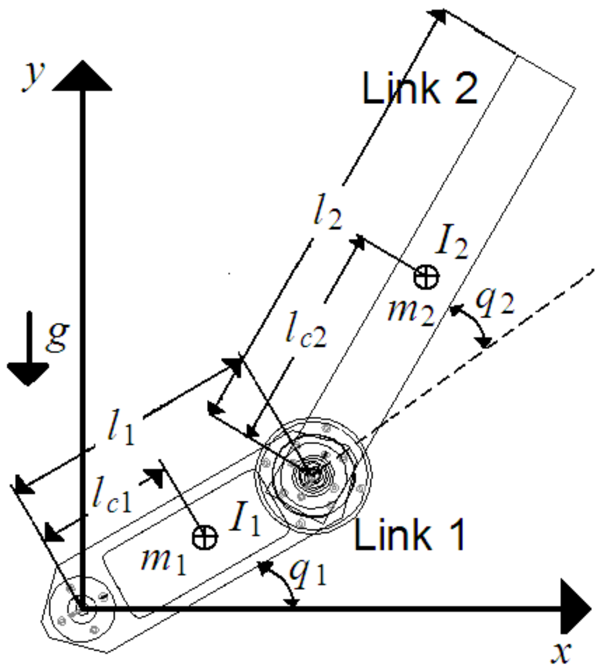

Fig. 1. schematic representation of the Pendubot

control at the right moment. This switching strategy consists in defining a neighborhood where the links are considered at the upwards position or in other words, in the basin of attraction of the lineal controller (LQR). When the links are in the neighborhood, the control law must be switched. In this paper, such a neighborhood is called the attraction region of the LQR.

The development of the dynamic equations of the Pendubot can be found in [9], which can be written as in (16), where $\tau = [\tau_1,\ 0]^T$ is the motor torque vector, $q = [q_1,q_2]^T, \dot{q} = [\dot{q}_1,\ \dot{q}_2]^T$ and $\ddot{q} = [\ddot{q}_1,\ \ddot{q}_2]^T$ are the position, velocity and acceleration vectors of the links 1 and 2, respectively.

$$\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) \tag{16}$$

Since in this work the nature of the open-loop control law is bang-bang and as the actuator (a DC armature controlled motor) can not deliver a perfect bang bang signal, the dynamics of the motor (the electrical constant time τ_e and the mechanical constant time τ_m) must be considered. Hence, the Pendubot dynamics (16) coupled to the motor dynamics is taken into account and it can be found in [13]. As it is stated in section 2, a nonlinear state representation of the system (2) is considered, where $x = [q_1,\ q_2,\ \dot{q}_1,\ \dot{q}_2,\ i_a]^T$ is the state vector, i_a is the armature motor current and $u(t) = [V_{in},\ 0]^T$ is the input voltage vector.

In Fig. 1, the schematic representation of the Pendubot is shown, where m_1,

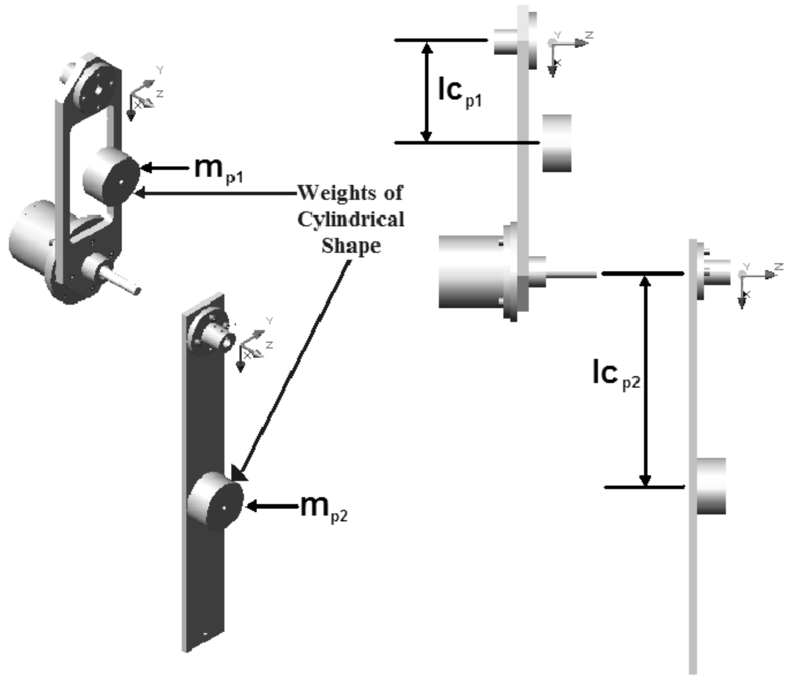

Fig. 2. CAD drawing of the Pendubot with the added cylindrical weights

m_2, l_1, l_2, l_{c1}, l_{c2}, I_1, I_2 are the masses, lengths, lengths between the rotation axis and the center of mass, and the inertias of links 1 and 2 respectively.

To implement the concurrent redesign approach, it is necessary to modify the structural parameters of the Pendubot. Then by considering that the Pendubot is already built, a weight of cylindrical shape is placed at each link in order to modify the dynamic parameters (m_i, l_{c_i}, I_i, $i = 1, 2$) of the Pendubot. These cylindrical weights are shown in Fig. 2, where m_{p1}, m_{p2}, lc_{p1}, lc_{p2} are the masses and the distance between the rotation axis and the mass center of cylindrical weights 1 and 2. Thus, $\Upsilon = [m_{p1}\ [Kg], m_{p2}\ [Kg], lc_{p1}\ [m], lc_{p2}\ [m]]^T \in R^4$ is the structural parameter vector to be modified.

The dynamic parameters of the Pendubot with the added cylindrical weights are obtained via classical methods [14], and summarized in Table 1. Note that these dynamic parameters ($\tilde{m}_i$, $\widetilde{lc}_i$, $\tilde{I}_i$, $i = 1, 2$) are functions of both the constant dynamic parameters of the Pendubot ($m_1 = 0.8435$ $[Kg]$, $m_2 = 0.3859$ $[Kg]$, $l_{c1} = 0.1573$ $[m]$, $l_{c2} = 0.1416$ $[m]$, $I_1 = 0.0057$ $[Kgm^2]$, $I_2 = 0.00539$ $[Kgm^2]$) and the parameters of the cylindrical weights which are grouped in Υ, where $Iz_{p1} = \frac{1}{2}m_{p1}(r_{p1})^2 + (m_{p1})(lc_{p1} - \widetilde{lc}_1)^2$ $[Kgm^2]$, $Iz_{p2} = \frac{1}{2}m_{p2}(r_{p2})^2 + m_{p2}(lc_{p2} - \widetilde{lc}_2)^2$ $[Kgm^2]$ and $r_{p1} = 0.0254$ $[m]$, $r_{p2} = 0.0254$ $[m]$ are the inertias and the radii of cylindrical weights on links 1 and 2 respectively.

It is important to note that changes in the structural parameters of the

Table 1
Dynamic parameters of the Pendubot with the cylindrical weights

Link 1	Link 2
$\tilde{m}_1 = 0.8435 + m_{p1}$ $[Kg]$	$\tilde{m}_2 = 0.3859 + m_{p2}$ $[Kg]$
$\widetilde{lc}_1 = 0.1573 + \frac{(lc_{p1})(m_{p1})}{\tilde{m}_1}$ $[m]$	$\widetilde{lc}_2 = 0.1416 + \frac{(lc_{p2})(m_{p2})}{\tilde{m}_2}$ $[m]$
$\tilde{I}_1 = 0.0057 + Iz_{p1}$ $[Kgm^2]$	$\tilde{I}_2 = 0.00539 + Iz_{p2}$ $[Kgm^2]$

cylindrical weights (Υ) modify the dynamic parameters of the whole system ($\tilde{m}_i$, $\widetilde{lc}_i$, $\tilde{I}_i$, $i = 1, 2$). Now, the motion equation of the Pendubot $\dot{x} = f(x(t),\Delta t,\Upsilon)$ is a function of the state vector $x(t)$, the switching time interval vector Δt of the control signal $u(t)$ and the structural parameter vector Υ of the cylindrical weights.

Changes in m_{pi} for $i = 1, 2$ must be positive because mass can only be added to the already constructed Pendubot and changes in lc_{pi} for $i = 1, 2$ could be positive or negative. A negative change in lc_{pi} means that the displacement of the cylindrical weight is on the $-x$ axis (see Fig. 2). Therefore, in order to establish the permissible changes in the structural parameter vector Υ of the cylindrical weights, the inequality constraint $\Upsilon_{min} \leqslant \Upsilon \leqslant \Upsilon_{max}$ (6) is introduced.

4 Simulation and experimental results

The swinging up problem of the redesigned Pendubot is solved via the concurrent redesign approach described in section 2. This approach simultaneously finds both the switching interval vector $\Delta t = [\Delta t_1 \cdots \Delta t_{n_{\Delta t}}] \in R^{n_{\Delta t}}$ and the structural parameters of the weights $\Upsilon = [m_{p1},\ m_{p2},\ lc_{p1},\ lc_{p2}]^T \in R^4$ that minimize $J(\Delta t, \Upsilon) \equiv t_f + \bar{\bar{\Xi}} = \sum_{j=1}^{n_{\Delta t}} |\Delta t_j| + \sum_{l=1}^{4} \underset{l}{\Xi}$ subject to the motion equation of the Pendubot-Motor $\dot{x} = f(x(t), \Delta t, \Upsilon)$, the initial position $x_o = [-\frac{\pi}{2}, 0, 0, 0, 0]^T$ and the final position $x_f = [\frac{\pi}{2}, 0, 0, 0, 0]^T$, assuming that there is a bang-bang control $u(t)$ defined in (7) where $u_{max} = 45$ $[volts]$ and $u_{min} = -45$ $[volts]$. The minimum and maximum limit vectors $\Upsilon_{min} = [m_{p1\,\min},\ m_{p2\,\min},\ lc_{p1\,\min},\ lc_{p2\,\min}]^T$ and $\Upsilon_{max} = [m_{p1\,\max},\ m_{p2\,\max},\ lc_{p1\,\max},\ lc_{p2\,\max}]^T$ are explicitly defined as $\Upsilon_{min} = [0,\ 0,\ -0.20,\ ,0.0381]^T$ and $\Upsilon_{max} = [0.2,\ 0.135,\ 0.127,\ 0.3048]^T$. These limit vectors were established by a trial and error procedure. This procedure consists of establishing an initial limit vector Υ_{min}, Υ_{max}. Then the DOP (8)-(11) is solved by the projected gradient algorithm (12) in order to find the optimum structural parameter $\Upsilon^{\star}$. If an optimum structural parameter $\Upsilon_i^{\star}$, $i = 1, \ldots, 4$ hits its limit value ($\Upsilon_i^{\star} = \Upsilon_{i\,\min}$ or $\Upsilon_i^{\star} = \Upsilon_{i\,\max}$), this limit value could be enlarged if and only if this is physically possible. Then, other optimization process which considers the modified limit vector will be required for finding the optimum structural parameter vector. This procedure is repeated until the optimum structural parameter vector $\Upsilon^\star$ does not hit the limit vector or even when it is not physically possible to enlarge the limit vector.

The constants of the barrier functions ω_Υ and the step sizes k_r, k_n required in (12), are selected as in (17), where "$diag(\bullet)$" is the diagonal matrix with the elements $(\bullet)$ in the diagonal.

$$\begin{aligned} \omega_{\Upsilon_l} &= 5E-8 \qquad\qquad\qquad for \quad l = 1 \quad to \quad 4 \\ k_r &= diag(1E-3, ..., k_{r_{5,5}}, 3E-4, 3E-4, 5E-4, 5E-4) \\ k_n &= diag(1E-4, ..., k_{n_{5,5}}, 1E-4, 1E-4, 1E-4, 1E-4) \end{aligned} \tag{17}$$

As the projected gradient algorithm could converge to a local minimum, several runs were performed in numerical simulations by using different starting conditions (Υ_{ini}, Δt_{ini}), which were randomly generated inside the feasible region imposed by Υ_{min} and Υ_{max}. The starting condition that generated the lowest value of the performance index (8) resulted as indicated in (18) and (19), with the optimal redesign structural parameters (20) and switching optimal intervals (21).

$$\Upsilon_{ini} = [0.10,\ 0.034,\ 0.05,\ 0.1]^T \tag{18}$$

$$\Delta t_{ini} = [-0.08,\ 0.65,\ -0.4,\ 0.1,\ -0.05]^T \tag{19}$$

$$\Upsilon^\star = [0.1990,\ 0.0003,\ -0.1425,\ 0.1532]^T \tag{20}$$

$$\Delta t^\star = [-0.029,\ 0.614,\ -0.356,\ 0.088,\ -0.007]^T \tag{21}$$

With the optimum parameters (20) and (21), the Pendubot brings the links from the rest position to the upwards position in a minimum final time of $t_f^\star = \sum_{j=1}^{5} \left|\Delta t_j^\star\right| = 1.09\ [s]$. In (20), it is important to note that the cylindrical weight of the link 2 should not be considered, as its mass $m_{p2}^\star$ is almost equal to zero. Then, the mass $m_{p1}^\star$ and the distance between the rotation axis and the center of mass $lc_{p1}^\star$ at the first link are the structural parameters which affect the positioning time of the Pendubot.

To implement the above results experimentally and since $lc_{p1}^\star$ is negative, a rectangular support with small mass is added in order to place the cylindrical

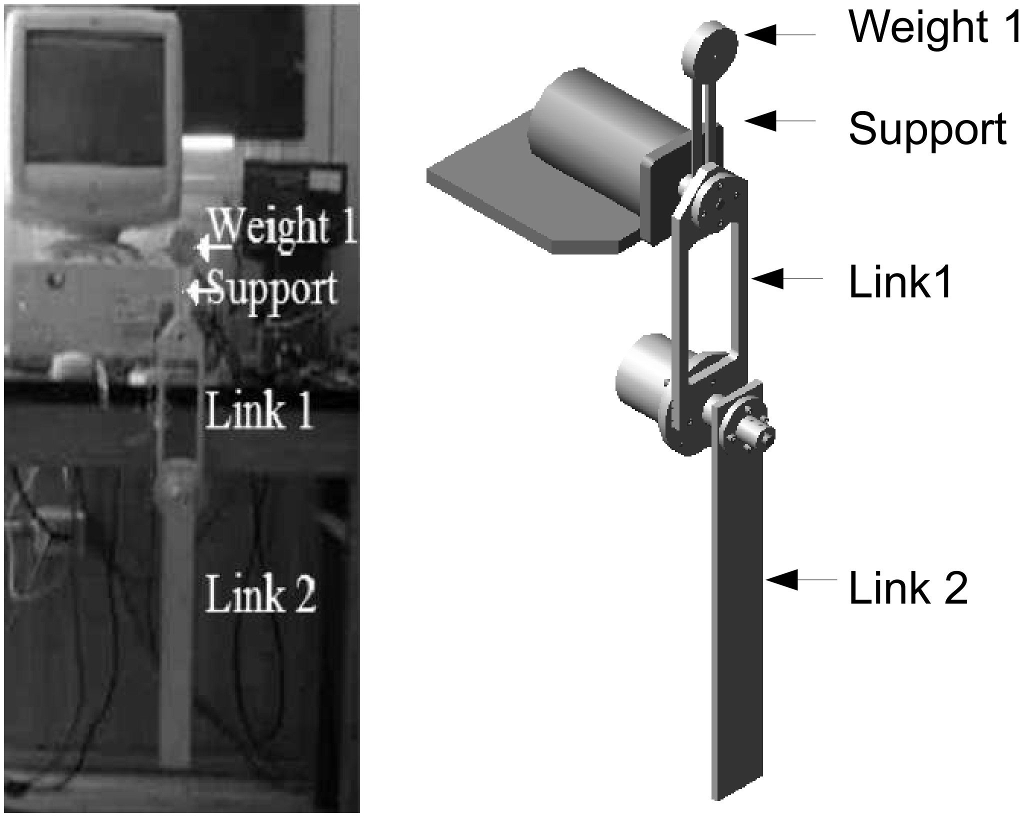

Fig. 3. Pendubot with the optimal structural parameters

weight of the first link on the direction $-x$ (Fig. 2). This support is made of aluminum with a mass of 0.035 $[Kg]$ and it is shown in Fig. 3.

A modeling error of the Pendubot can produce nonoptimal design parameters for the real system, which could not appropriately carry out the positioning task. Therefore, the optimization algorithm is computed again in a series of experimental trials, but now with the state trajectory measured from the real system instead of the computed trajectory. Hence, once we have found the optimal simulation results, the gradient projection algorithm (12) must simultaneously compute both the optimal switching interval vector Δt^c_{new} and the optimal structural parameter Υ^c_{new} for the experimental results, but now with the real state trajectory and the real final state error. This procedure could be repeated if the terminal state requirement $e_f \approx 0$ is not fulfilled. As a result, the optimal parameters for the experimental setup are obtained. With this procedure, the switching interval vector Δt^c_{new} is the only that significatively changes. As opposite to the structural parameters whose changes were infinitesimally small, that is, $\Upsilon^c_{new} \approx \Upsilon^\star$ which were not physically feasible. The optimal switching interval vector for the experimental setup is shown in (22), where $\Delta t^{c\star}$ and $\Upsilon^{c\star}$ represent the optimal design parameters for the experimental results.

$$\Upsilon^{c\star} = [0.1990, \;\; 0.0003, \;\; -0.1425, \;\; 0.1532]^T$$
$$\Delta t^{c\star} = [-0.07, \;\; 0.58, \;\; -0.245, \;\; 0.06, \;\; -0.001]^T \tag{22}$$

The optimal switching interval vector and the optimal structural parameters (22) for the experimental setup produced a swinging up time $t_f^{c\star} = \sum_{j=1}^{5} \left|\Delta t_j^{c\star}\right| = 0.954$ $[s]$.

The objective of the concurrent redesign approach (C. R. approach) is to adjust the structure-control parameters in order to bring the system from one place to another in minimum time, but when the system is in an environment with external forces, specially the gravity force, the concurrent redesign approach does not guarantee that the system permanently stays in the last position. So, a LQR control is used in order to stabilize the state vector $x(t)$ at the state $\bar{x} = x(t_f) = \left[\frac{\pi}{2}, 0, 0, 0, 0\right]^T$. The LQR gains K_{p-m} are obtained by using a linearized model of the Pendubot around the state $\bar{x}$ and the "*lqrd*" Matlab function. Hence, $K_{p-m} = [-403.5333, -421.7918, -91.8461, -64.0117, 2.9301]$ are the optimal LQR gains.

The switching of the control law from the one obtained by the C. R. approach to the LQR, is carried out when the links are in the attraction region of the LQR, given by, $|e_{x_1}| \leq 0.139\,63$ $[rad]$, $|e_{x_2}| \leq 0.104\,72$ $[rad]$, $|e_{x_3}| \leq 4$ $[\frac{rad}{s}]$ and $|e_{x_4}| \leq 8$ $[\frac{rad}{s}]$, where $e_{x_i} = x_i(t) - x_{f_i}$, which was experimentally obtained as the one that minimizes the norm of the attraction region, given by the term $\sqrt{e_{x_1}^2 + e_{x_2}^2 + e_{x_3}^2 + e_{x_4}^2}$. Different attraction regions would lead to different experimental positioning times. However, we are interested in the attraction region allowing the Pendubot to stay the longer time being controlled by the bang-bang control instead the LQR control.

In Fig. 4, we show the experimental results of the C. R approach. At the first second, the open-loop bang-bang control $u(\Delta t)$ with the optimal switching interval vector (22) is applied. At time $t = 0.895$ $[s]$, the states of the Pendubot are inside the attraction region, then the LQR control replaces the bang-bang control, stabilizing the links at $\bar{x}$. Then the required time to switch the control law is 0.895 $[s]$. Due to the attraction region of the LQR, the final time in this experiment is shorter than $t_f^{c\star}$.

In Table 2 the summary of the simulation and experimental results of the concurrent redesign approach is shown. The final time of experimental results is enhanced because the optimal design parameters $\Delta t^{c\star}$ and $\Upsilon^{c\star}$ were modified when the gradient projection algorithm was computed again with the state trajectory feedback from the real system, instead of the simulated trajectory. It should be pointed out that the experiment was conducted several times. In order to give a statistical measure of the solution behavior, a fifty sample set provided an average final time of $t_f = 0.895$ $[s]$ and a standard deviation of 0.0204 $[s]$.

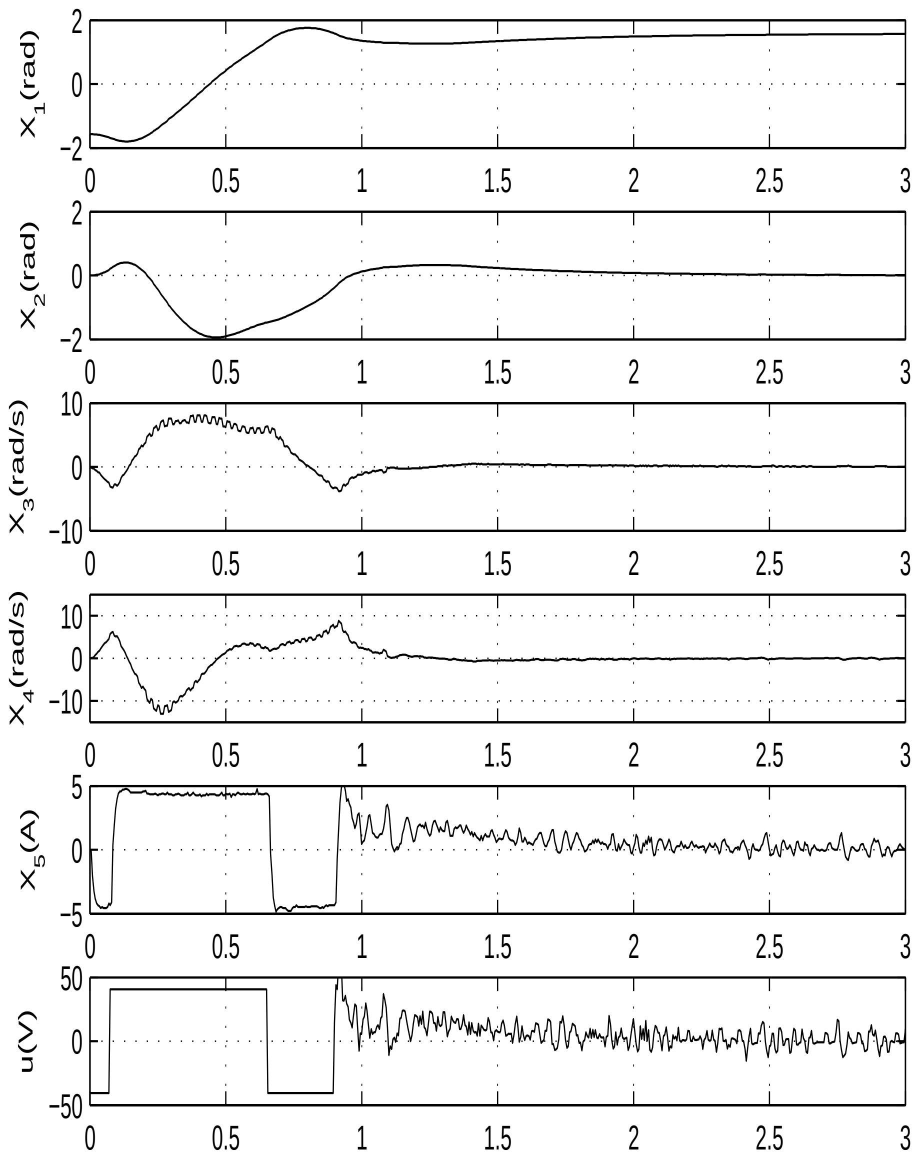

Fig. 4. Experimental results to swing up and stabilize the Pendubot with the concurrent redesign approach and a LQR control respectively

Table 2
The simulation and experimental results of the concurrent redesign approach

	C. R. Approach (t_f)
Simulation Results	1.09 [s]
Experimental Results without LQR	0.954 [s]
Experimental Results with LQR	0.895 [s]

5 Conclusion

In this paper, a concurrent redesign approach that involves structural and a bang-bang control law parameters is proposed. This approach reformulates the concurrent redesign problem as a dynamic optimization problem, where the independent variables are the structural (mechanical) and control parameters of an electromechanical system. To solve such a dynamic optimization problem, the projected gradient algorithm is used, therefore, all functions involved in the redesign process, should be at least once differentiable.

The concurrent redesign approach has been illustrated with a highly nonlinear underactuated double pendulum system called the Pendubot.

A weight of cylindrical shape is placed at each link in order to change the dynamic parameters ($m_1,\ m_2, l_{c1},\ l_{c2},\ I_1,\ I_2$) of the Pendubot. Then, the structural parameters to be modified are the masses m_{p1}, m_{p2} and the distances between the rotation axis and the center of mass $lc_{p1},\ lc_{p2}$ of the weights of cylindrical shape.

The concurrent redesign approach finds the switching interval vector of the bang bang control law and the structural parameters of the weights of cylindrical shape in order to minimize the required time to bring the redesign Pendubot from the rest position $x(t_0) = x_0$ to the upward position $x(t_f) = x_f$ in minimum time.

If the system has already been built or not, with this concurrent redesign approach it is possible to improve the performance of an electromechanical system. When the system has already been built, it is necessary to add a subsystem in order to modify the structural parameters of the original system. When the system is not constructed, it is necessary to select the shape (structure) of the system and hence the structural parameters that the designer wishes to optimize.

Although the concurrent redesign approach is illustrated with an underactuated manipulator, it is considered that this approach can be easily extended to other kinds of electromechanical systems. Specially, for redundant manipulators, new configurations could be obtained by avoiding obstacles or state constraints.

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