ARTICLE TEMPLATE

Meta-heuristic algorithms for the control tuning of omnidirectional mobile robots

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ABSTRACT

The increment of efficient mobile robots in Engineering has motivated the search for new alternatives to improve the control tuning task. In this paper, the Cartesian space Proportional-Derivative control tuning for Omnidirectional Mobile Robots is established under an offline dynamic optimization approach where the minimization of the tracking error and energy consumption are simultaneously considered providing an efficient performance in the real test. The statistical study on the performance of twelve different meta-heuristic algorithms and one gradient technique indicates that using the fittest solution in the meta-heuristic optimization process through generations allows finding more suitable controller parameters. Also, real tests with each of the best control gains obtained through algorithms are performed into a laboratory prototype. The laboratory test analysis statistically indicates that the 75% of comparisons with the best control gains present different performance functions in spite of presenting slightly different control gains.

KEYWORDS

Optimum control tuning; Omnidirectional Mobile Robot; Optimization methods; Meta-heuristic algorithms; Cartesian space Proportional-Derivative control

1. Introduction

In recent years, mobile robot control has received wide attention and is a topic of great research interest due to a large number of situations that humans cannot accomplish, and the practical importance of its applications in spatial or terrestrial reconnaissance (Vallvé and Andrade-Cetto 2015), in the execution of tasks in hard environments (Zhao et al. 2016), in material handling (Peng et al. 2016), in cooperative tasks (Baca et al. 2015), among others. Thus, the efficiency increment in robot movements has become more significant over time.

Proportional-Integral-Derivative (PID), Proportional-Integral (PI) and Proportional-Derivative (PD) controllers have been widely used in industrial applications instead of applying advanced control strategies (Khalil 2014) due to their simplicity and its ease of implementation. Moreover, the PID controller is one of the most used low-level control strategies, and its response depends on the setup of their parameters. Therefore, finding suitable control parameters for the mobile robot is a

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crucial task to increase the performance of the control system (Reynoso-Meza et al. 2009).

The study of the PID control tuning began in 1942 where empirical procedures were established to regulate linear systems (Ziegler and Nichols 1942). Nowadays, the use of computational intelligence and soft computing to rule-based and knowledge-based system methodologies in the control engineering area have increased recently. The use of such methodologies in control engineering is generally referred to as Intelligent Control (Ruano 2007), and one of the most important problems in this research area is the control tuning. Among control tuning methodologies (Villarreal-Cervantes and Alvarez-Gallegos 2016), the optimization methods and the adaptive methods are gaining more importance due to the necessity for increasing the overall performance of machines at the same time that a set of requirements are met. These tuning methodologies are established as a mathematical programming problem where the most appropriate control gains are found offline (optimization methods) or online (adaptive methods) based on a cost function (Caponio et al. 2007). Meta-heuristic algorithms, whose operation is inspired by the behavior of natural systems, are used more and more frequently to solve optimization problems in the design of mechatronic systems with a high degree of synergy in the structure-control design framework (Villarreal-Cervantes 2017; Portilla-Flores et al. 2011); in robotic applications for complex tasks (Fong, Deb, and Chaudhary 2015) such as path planning (Mac et al. 2016) in unknown environments (Kulich, Miranda-Bront, and Přeučil 2017), the gait generation of humanoid robots (Huan et al. 2018), among others; and in the adjustment of controller parameters for mechatronics systems using optimization (Villarreal-Cervantes and Alvarez-Gallegos 2016) and adaptive (Villarreal-Cervantes et al. 2017; Villarreal-Cervantes, Mezura-Montes, and Guzmán-Gaspar 2018) tuning methods. The above because of the ability of these algorithms to find a solution near the global one in highly nonlinear, discontinuous or non-convex design spaces, and because they do not require additional problem information such as gradients, Hessian matrices, initial search points, etc. Moreover, they are independent of the problem characteristics, i.e., they can be used/adapted to solve a wide variety of problems from different contexts. Nevertheless, statistical tests must be developed to validate the performance of the obtained results due to the stochastic way in which these algorithms search for solutions (Derrac et al. 2011). Also, it can be necessary for these algorithms to be computationally efficient for particular problems such as the control tuning based on adaptive methods, where the computational burden must be minimized to allow the online computing of the best controller parameters (Rodríguez-Molina, Villarreal-Cervantes, and Aldape-Pérez 2017; Rodríguez-Molina et al. 2019).

In this paper, we are interested in offline control tuning based on optimization methods. The above because these methods do not require a high computational cost to the real application, since once the optimum control gains are found, they are implanted in the real control system and remain fixed (no additional changes are required in the closed-loop system), which in turn can be easily implemented for an industrial robot. In this sense, the real implementation (laboratory test) is one of the main issues presented in such methods, where an important lack of real experimental results is detected which are considered indispensable to show the reliability of this tuning approach. One example of the efficiency of meta-heuristic algorithms in the controller tuning based on optimization methods is shown in (Jiménez et al. 2015), where a genetic algorithm is used to automate the tuning process of Passive Optical Networks and up to 64% of the tuning time is reduced. In (Amador-Angulo and Castillo 2018), the Bee Colony Optimization (BCO) algorithm is used to tune a type-1 fuzzy logic controller for two benchmark control problems: the water tank regulation control and the mobile robot trajectory tracking. The fuzzy dynamic adaptation is included in the optimal search of two parameters of the BCO algorithm allowing this last to run more precisely and efficiently. The type-2 fuzzy controllers for autonomous mobile robots are tuned by using the firefly algorithm in (Lagunes et al. 2018). Similar to (Amador-Angulo and Castillo 2018), the dynamic adjustment of the firefly algorithm is carried out by using type-1 and type-2 fuzzy logic showing superior results.

On the other hand, some enhancement strategies have been incorporated in meta-heuristic algorithms as in (Wei and Shun 2010), where the velocity equation of the Particle Swarm Optimization (PSO) is updated and hence the tracking performance of the PID controller for a pendulum system is improved by using the obtained optimal controller gains. Another research is the work proposed in (Helon and Leandro 2012), which provides useful information in simulation about the tuning process of the PID controller for a two degree of freedom ($d.o.f.$) robotic system. In this, a multi-objective optimization problem is developed, and the non-dominated sorting genetic algorithm II is used to find suitable trade-offs between the position error and the smoothness of the control signal. In (Pan, Das, and Gupta 2011), a comparative closed-loop performance evaluation for the optimal PID and the optimal fuzzy PID based networked control system is presented. These optimal control gains are obtained through four variants of the PSO and then are tested in simulation such that superior performance is given by the lbest PSO variant. Robustness in the PID control gains, which are found with the PSO algorithm, is obtained for three hydraulic tank systems in (Karer and Škrjanc 2016), through considering sensitivity function constraints into the optimization problem. Simulation results confirm that those gains were useful when system perturbations had not been considered in the design phase, as well as the case where linear systems are only considered into the optimization process instead of nonlinear ones. Nevertheless, in a real application (laboratory tests with an experimental prototype), several nonlinear uncertainties such as noise in the input/output signals, backlash, etc., are not included into the optimization problem due to they may not be known in advance. In previous works, the efficiency of the control tuning strategy was verified through a simulated environment. In the experimental environment (test with a real prototype), the unmodeled uncertainties (unmodeled dynamics) can deteriorate the closed-loop system performance provided by the obtained control gains (Karer and Škrjanc 2016). Those uncertainties are usually unknown such that the obtained control gains may not be for the real environment or they would require an additional adjustment to provide high control performance. Hence, to the best of the author's knowledge, there is no formal analysis that shows enough evidence of its use in practice of the use of the control tuning based on optimization methods as an offline dynamic optimization approach for real Omnidirectional Mobile Robots (OMRs).

One of the main characteristics of the OMR is the ability to accomplish linear and angular movements simultaneously. Such robots present highly nonlinear and coupled behaviors such that intelligent control system methodologies based on an optimization process and meta-heuristic algorithms are promising research fields to handle such behaviors and perform successful operations in mobile robots (Carlucho et al. 2017). Nevertheless, a formal statistical comparative analysis of such methodologies in the control tuning of the OMRs based on optimization methods and meta-heuristic algorithms has not been developed. Moreover, the narrow differences in the control gains obtained through different algorithms in an offline dynamic optimization approach for optimal tuning of the OMR control implemented in a laboratory prototype, have not been analyzed. This analysis indicates the effects of different optimal control gains on control performance changes in a real environment.

Hence, the main contributions of this work are: *i*) The proposal of the control tuning based on an optimization method as an offline dynamic optimization approach. This includes the formulation of the single-objective optimization problem whose solution is a set of control parameters that provide a good trade-off between trajectory tracking performance and energy consumption. The obtained optimum control gains in the control system not only perform a specific trajectory, but also other elemental curves with an outstanding performance. *ii*) The statistical study of the effectiveness of twelve different meta-heuristic algorithms, and one gradient-based technique. This study is used to identify promising alternatives or beneficial meta-heuristic mechanisms that aid the improvement of the Omnidirectional Mobile Robot control performance. *iii*) The empirical analysis of experimental results with the control gains obtained by each algorithm to know the benefits on the performance function of the real Omnidirectional Mobile Robot when slight variations on the control gains are presented. Moreover, this analysis confirms the effectiveness of the proposed tuning approach where additional comparisons with the gain-scheduled LQR show the reliability of the proposal.

The rest of the paper is organized as follows. Section 2 introduces the way to formulate the control tuning problem as a methodology based on optimization methods where the performance function and constraints are proposed. Also, the mathematical description of the mobile robot is explained for the inclusion of the dynamic constraint, and also the PD control system is introduced. A brief description of the different meta-heuristic algorithms and the gradient-based one, used to tune the control system is presented in Section 3. Section 4 shows the comparative analysis of optimizers based on simulation and experimental results. The conclusions are drawn in Section 5.

2. Offline dynamic optimization approach for optimal tuning of the OMR control

The schematic diagram of the OMR is shown in Fig. 1, where the mass and the inertia of the mobile robot are represented by m and I_z; r and J are the radius and the inertia of wheels; L is the distance between the mass center of the mobile robot and wheels; θ_i and $\dot{\theta}_i$ are the angular position and velocity of the $i-th$ wheel. The state vector corresponding to the linear/angular position and velocity of the Omnidirectional Mobile Robot expressed in the inertial coordinate system is denoted by $x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = \left[x_w, y_w, \phi_w, \dot{x}_w, \dot{y}_w, \dot{\phi}_w\right]^T \in R^6$. The dynamics of the OMR is given in (1), where the used Cartesian space PD control is expressed in (4) considering $e = \left[\bar{x}_d - x_1, \bar{y}_d - x_2, \bar{\phi}_d - x_3\right]^T$ and $\dot{e} = \left[\dot{\bar{x}}_d - x_4, \dot{\bar{y}}_d - x_5, \dot{\bar{\phi}}_d - x_6\right]^T$ as the lineal/angular position and velocity error vectors between mobile position/velocity and the desired ones $\bar{x} = \left[\bar{x}_d, \bar{y}_d, \bar{\phi}_d, \dot{\bar{x}}_d, \dot{\bar{y}}_d, \dot{\bar{\phi}}_d\right]^T$ and control gains are represented as $k_p = diag\left(k_{p1}, k_{p2}, k_{p3}\right) \in R^{3x3}$ and $k_d = diag\left(k_{d1}, k_{d2}, k_{d3}\right) \in R^{3\times3}$.

$$\dot{x} = f(x,p) + g(x,p)u \tag{1}$$

where the terms in (1) are defined in (2)-(3) considering $\vartheta = \frac{r}{2mr^2+3J}$,

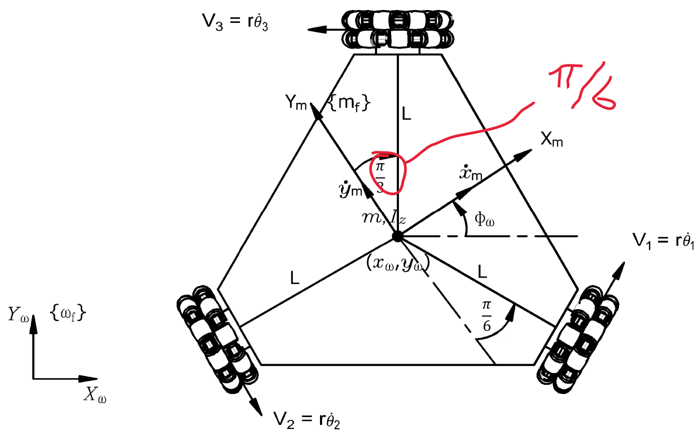


Figure 1. Schematic diagram of the OMR.

$$f(x,p) = \begin{bmatrix} x_4 & x_5 & x_6 & -\frac{3J}{2mr^2+3J}x_6x_5 & \frac{3J}{2mr^2+3J}x_4x_6 & 0 \end{bmatrix}^T \tag{2}$$

$$g(x,p) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\vartheta\left(\sin x_3 - \sqrt{3}\cos x_3\right) & 2\vartheta \sin x_3 & -\vartheta\left(\sin x_3 + \sqrt{3}\cos x_3\right) \\ \vartheta\left(\cos x_3 + \sqrt{3}\sin x_3\right) & -2\vartheta\cos x_3 & \vartheta\left(\cos x_3 - \sqrt{3}\sin x_3\right) \\ \frac{Lr}{3JL^2+I_zr^2} & \frac{Lr}{3JL^2+I_zr^2} & \frac{Lr}{3JL^2+I_zr^2} \end{bmatrix} \tag{3}$$

and the controller u is given by (4),

$$u = \breve{J}^T(k_pe + k_d\dot{e}) \tag{4}$$

considering the Jacobian matrix $\breve{J} \in R^{3\times 3}$ in 5.

$$\breve{J} = \begin{bmatrix} \frac{-\sin x_3 + \sqrt{3}\cos x_3}{3} & \frac{2}{3}\sin\phi & -\frac{\sin x_3+\sqrt{3}\cos x_3}{3} \\ \frac{\cos x_3+\sqrt{3}\sin x_3}{3} & -\frac{2}{3}\cos\phi & \frac{\cos x_3 - \sqrt{3}\sin x_3}{3} \\ \frac{1}{3L} & \frac{1}{3L} & \frac{1}{3L} \end{bmatrix} \tag{5}$$

Let the design variable vector $p = [k_{p1}, k_{p2}, k_{p3}, k_{d1}, k_{d2}, k_{d3}]^T \in R^6$ be the control gains, the offline dynamic optimization approach to the optimal tuning of the Omnidirectional Mobile Control aims to find the optimal vector p^* which improves the performance of both the accuracy of the trajectory tracking $\bar{J}_1 = \int_0^{tf} e_1^2 dt + \int_0^{tf} e_2^2 dt + L^2 \int_0^{tf} e_3^2 dt \in R$ (related to the sum of the squared Cartesian position error in $X_w - Y_w$ between the desired position and the position generated by the mobile robot) and the energy consumption $\bar{J}_2 = \int_0^{tf} u_1^2 dt + \int_0^{tf} u_2^2 dt + \int_0^{tf} u_3^2 dt \in R$ of the Cartesian space controller $u = [u_1, u_2, u_3]^T \in R^3$ (related to the square of the total torque applied by the control system to the mobile robot). Also, this is subject to the natural dynamic constraints of the OMR (7), the desired trajectory to be fulfilled (8)-(10), with $f = 1/60$ and $f_\phi = 1/120$, and the search space bounded by the minimum and maximum values of the design variable vector (11) denoted by p_{min} and p_{max}, respectively. At the beginning of the trajectory (8)-(10), the mobile robot must position itself smoothly at the initial point of a hypocycloid path in the time $t = 10s$. The smooth trajectory is given by a Bézier polynomial $\varphi = \Delta^5(126 - 420\Delta + 540\Delta^2 - 315\Delta^3 + 70\Delta^4)$, where $\Delta = t/10$. After that time ($t \geq 10s$), the hypocycloid trajectory is generated. This assembled trajectory is chosen because it presents highly nonlinear dynamics where linear controllers can display some issues in the trajectory tracking by the mobile robot.

In this paper the multi-objective optimization problem is transformed to a single-objective optimization problem by using the weighted sum approach (Osyczka 1984) in order to fulfill one trade-off of the Pareto front. Such trade-off is a priori selected by the weights μ_1 and μ_2. The formal mathematical dynamic optimization problem is stated in (6)-(11), where the aggregation function $\bar{J} = \mu_1\bar{J}_1 + \mu_2\bar{J}_2$ weights both criteria ($\bar{J}_1$ and $\bar{J}_2$). In this case, the weights $\mu_1 = 0.95$ and $\mu_2 = 0.05$ are chosen as those where an appropriate performance in the trajectory tracking and the energy consumption (trade-off) is developed by the control system of the mobile robot. Those weights were found though a rigourous trial and error procedure in order to fulfill the specific trade-off between both criteria.

$$\underset{p^* \in R^6}{Min} \ \bar{J} \tag{6}$$

subject to:

$$\frac{dx}{dt} = f(x, p) + g(x, p)u \tag{7}$$

$$h_1 : \bar{x}_d - \begin{cases} \varphi & \forall\, t \leq 10 \\ 0.8181\cos(2\pi f t) + 0.1818\cos(9\pi f t) & \forall\, t > 10 \end{cases} = 0 \tag{8}$$

$$h_2 : \bar{y}_d - \begin{cases} 0 & \forall\, t \leq 10 \\ 0.8181\sin(2\pi f t) - 0.1818\sin(9\pi f t) & \forall\, t > 10 \end{cases} = 0 \tag{9}$$

$$h_3 : \bar{\phi}_d - \begin{cases} 0.4363\varphi & \forall\, t \leq 10 \\ 0.4363\cos(2\pi f_\phi t) & \forall\, t > 10 \end{cases} = 0 \tag{10}$$

$$p_{min} \leq p \leq p_{max} \tag{11}$$

3. Optimization algorithms

The problem in (6)-(11) is a constrained highly-nonlinear single-objective dynamic optimization problem. Solving a single-objective problem refers to finding the global minimum. In a feasible search space denoted by Ω (where all constraints are met), the global minimum is defined as the smallest value of the objective function $\bar{J}$ which is reachable only with the optimal solution $p^* \in \Omega$, i.e., $\bar{J}(p^*) \leq \bar{J}(p), \ \forall\, p \in \Omega$.

For the OMR optimal control tuning problem, p^* is the vector of the best controller parameters (proportional and derivative gains) for which the linear/angular position and velocity errors of the OMR, as well as the energy consumption, are as minimal as desired (depending on the *a priori* preference articulation given by the weights μ_1 and μ_2).

Meta-heuristics are techniques that can solve complex optimization problems such as the OMR optimal control tuning problem at a reasonable computational cost. They are mostly population-based stochastic algorithms that can find good solutions (near to the optimal solution) to different problems and are commonly inspired by natural processes.

When solving real-world problems, there is no *a priori* information about the location of the optimal solution or the shape of the search space. The problem could also be multi-modal (i.e., there are different vectors $p^* \in \Omega$ that reach the global minimum, or there are several local peaks around the global minimum) or the feasible space Ω may be very hard to find. For this reason, it is not possible to opt for a single algorithm to solve a particular problem of this type and ensure that it can get the best performance (Wolpert and Macready 1997).

In the last decades, meta-heuristic algorithms have been used to tune control systems in complex engineering problems based on optimization methods (Fleming and Purshouse 2002) and then, have been taken as a useful tool in the control community. Some of the most popular are the Differential Evolution (DE) algorithm (Das and Suganthan 2011; Das, Mullick, and Suganthan 2016), the Genetic Algorithm (GA) (Whitley and Sutton 2012), the Particle Swarm Optimization (PSO) (Zhang, Wang, and Ji 2015), the Firefly algorithm (FA) (Fister et al. 2013), and the Bat Algorithm (BA) (Chawla and Duhan 2015) . The above because they are easy to implement, are flexible (they can be modified in order to solve different kinds of optimization problems, e.g., continuous, discrete, nonlinear and, non-convex), perform well in terms of accuracy, convergence speed (solutions are obtained using a reasonable amount of computational time), and robustness in several kinds of benchmark and real-world problems.

In this work, such well-known meta-heuristic algorithms are adopted to solve the PD controller optimal tuning problem for the OMR. These meta-heuristics are: Differential Evolution (using the eight DE variants Rand 1 Bin, DE Rand 1 Exp, DE Best 1 Bin, DE Best 1 Exp, DE Current to Rand 1, DE Current to Best 1, DE Current to Rand 1 Bin and DE Current to Best 1 Bin), Genetic Algorithm, the Bat Algorithm, the Firefly Algorithm and the Particle Swarm Optimization. Additionally, the deterministic method of Sequential Quadratic Programming (SQP) (which is a gradient-based technique) is used to perform comparisons.

4. Results

In this section, the obtained results of the algorithm performance for the offline Omnidirectional Mobile Robot control tuning based on the dynamic optimization approach are analyzed via non-parametric statistical tests (Derrac et al. 2011). One gradient algorithm and twelve meta-heuristic techniques are chosen for such analysis. Moreover, the control performance in the OMR (measured from a laboratory test with the real prototype) with the optimum gains obtained from all techniques is discussed.

4.1. *Performance analysis of algorithms*

Thirty independent runs are carried out for each meta-heuristic technique, and one hundred independent runs are executed for the SQP algorithm. More runs with random initial conditions were proposed for the SQP algorithm due to the initial condition sensitivity which results in the convergence to local minima. The Friedman test is used to conclude about the algorithm performance.

Table 1. Parameters of the OMR. (Peñaloza-Mejía et al. 2015).

Parameter	*Description*	*Value*	*Units*
r	Wheel radius	0.0625	m
L	Wheel distance to the mass center	0.2870	m
m	Mass	16.319	kg
J	Wheel inertia	$5.82e-4$	$kg \cdot m^2$
Iz	Mobile robot inertia	0.5160	$kg \cdot m^2$

The conditions of the optimization problem are set as follows: Through the Lyapunov analysis of the Cartesian space PD control (Siciliano et al. 2009), the lower bound vector for the design variables $p_{min} = [0, 0, 0, 0, 0, 0]^T$ is selected. The upper bound vector $p_{max} = [5000, 5500, 3000, 25, 25, 100]^T$ is chosen based on the procedure given in (Villarreal-Cervantes and Alvarez-Gallegos 2016). The upper bounds in the derivative gains allows to link the simulation results to the experimental ones i.e., the obtained control gains in simulation can be successfully implemented in the real scenario in spite of the noise in the velocity estimation of the mobile robot. If those bounds (specifically in the derivative gains) are not considered into the optimization problem, the control system with the obtained gains can not follow the desired trajectory in the real experiment. The used nominal values for the kinematic and dynamic parameters of the OMR are shown in Table 1 and obtained through a Computer-Aided Design software; the uncertainties related to those parameters can ascend up to 3% due to the laboratory conditions. The Euler integration is employed in the simulation results to solve the dynamics of the OMR (1) with an initial condition $x^0 = \mathbf{0} \in R^6$, the integration time $\Delta t = 5ms$ and the final time $t_f = 130s$.

The algorithm conditions are selected as follows: All meta-heuristic optimizers have the maximum iteration number $G_{max} = 200$ as the stop criterion and the same population size $NP = 50$. It is important to note that the selected generation number G_{max} was obtained after a series of trials. These trials consist on increasing the generation number for all algorithms and select the maximum generation where the performance function $\bar{J}$ stays around a stable value. The other parameters of meta-heuristic algorithms are chosen by proposing a series of trial and error procedures around of the recommended values of the state of the art. In the case of SQP, an additional stop criterion is included because of its premature convergence to local minimum solutions. This criterion is the step length tolerance ($\|\lambda\| < 1E^{-15}$). In the case of differential evolution variants, the scale factor F is randomly selected between the interval $[0.3, 0.9]$ at each generation and the crossover rate is $CR = 0.6$ for all runs. The scale factor is set randomly between a specific interval to endow robustness into the DE variants (Iacca, Neri, and Mininno 2012). In the case of the GA, the selected parameters are $pm = 0.166$, $pc = 0.9$, $\eta_c = 20$ and $\eta_m = 20$. The parameters of the BA are: $f_{min} = 0.1$, $f_{max} = 0.9$, $A_i = 0.6$, $\alpha = 0.998$, $r_i^0 = 0.35$ and $\gamma \to \infty$ (i.e. $r_i = r_i^0$), with $i = 1, \ldots, NP$. The FA has the following parameters: $\alpha = 0.2$, $\beta_{min} = 0.2$, $\beta_0 = 1$, $\gamma = 1$ and $w = 0.95$. For the PSO algorithm the parameters are: $\vec{v}_{min} = -\frac{1}{2}\vec{p}_{min}$, $\vec{v}_{max} = \frac{1}{2}\vec{p}_{max}$, $C_1 = 1.6$ and $C_2 = 1.4$. In the case of the SQP algorithm, the initial solution is randomly generated in the interval $p_{min} \leq p_0 \leq p_{max}$ and the SQP was implemented using the function $fmincon$ of $MATLAB^{®}$.

In Table 2, the descriptive statistics about runs per each algorithm is presented. One sample of the descriptive statistics is related to the mean of the performance function of solutions in the last generation per each run of the meta-heuristic algorithm, and in the case of the SQP algorithm, one sample is related to the obtained result of each run. The second and third column of Table 2 contain the mean and the standard deviation of the samples. In the fourth and fifth columns, the mean of the maximum (the worst) and minimum (the best) performance function value of individuals in the last generation for all runs is given. In the sixth column, the deviation percentage between the mean best individual (fifth column) for each algorithm from the best result given in the same column (specified by DE Best 1 Bin $mean(\bar{J}_{best}) = 8.57936921e-3$) is shown. In order to know how many of the runs reach the optimal solution, the percentage of best individuals per each run that stay in a neighborhood of $8.57e-4\%$ from the global solution is presented in the last column ($\%BI$) of the same table. This small neighborhood is selected since we are interested in finding the optimal gains of the control system which carry out the most accurate trajectory tracking with the lowest energy consumption. On the other hand, the best results are remarked in boldface.

Different findings are observed in Table 2 and these are summarized as follows:

- The most competitive algorithms are given by DE Best 1 Bin and DE Best 1 Exp because they obtain the smallest value of the performance function (see columns 2 and 5), and also in all runs find an individual in the neighborhood of the global solution (see column 7). This indicates that exploiting the best individual generated through generations promotes the search for promising solutions.
- All swarm based optimization algorithms such as PSO, BA and FA present bad convergence of solutions presented in the last generation (see third column). This is attributed to the lack of a selection process into the search mechanism which is the main characteristic of those swarm based techniques. In spite of such bad convergence, some few solutions into the swarm converge to a good solution as can be seen in the fifth column.
- It is clear that the gradient-based algorithm (SQP) does not perform well in such problems and also presents high sensitivity to initial conditions because only four runs from a total of one hundred runs converge to local solutions (the rest of runs diverge). This indicates that the problem is multi-modal and hence the obtained solution with this algorithm hardly reach the global one with the chosen initial conditions.
- It is observed that the most reliable algorithms in the search for the global solution are DE Best 1 Bin, DE Best 1 Exp, DE Rand 1 Bin, DE Rand 1 Exp and GA because all runs reach the neighborhood of the global solution (see last column).
- Another important fact is that in spite of presenting small performance differences among algorithms (see column 6), for highly accurate applications in Small-Scale Robotics (Paprotny and Bergbreiter 2014), these differences could be very large. Therefore, the importance of this study lies in knowing the behavior of different optimizers in the control gain tuning for Omnidirectional Mobile Robots.

Non-parametric inferential statistical tests (Derrac et al. 2011) are applied to the results obtained in the samples of runs. As a first step, the 95%-confidence Friedman test is applied. According to the returned p-value ($9.0311e-53$) shown in Table S1, significant differences among meta-heuristic algorithm runs are observed and the ranks of the Friedman test indicate that DE Best 1 Bin is the best performing algorithm of the comparison with a rank of 1. Then, with the purpose of drawing general conclusions about the algorithm performance and finding accurate pairwise comparisons, the Friedman test for multiple comparisons with the most representative post-hoc error correction methods (Derrac et al. 2011) is presented in Table S2. Also, Table 3 sum-

Table 2. Descriptive statistical summary of the algorithm behavior. DE-R1B: DE Rand 1 Bin, DE-R1E: DE Rand 1 Bin, DE-B1B: DE Best 1 Bin, DE-B1E: DE Best 1 Exp, DE-CR1: DE Current to Rand 1, DE-CB1: Current to Best 1, DE-CR1B: Current to Rand 1 Bin, DE-CB1B: Current to Best 1 Bin.

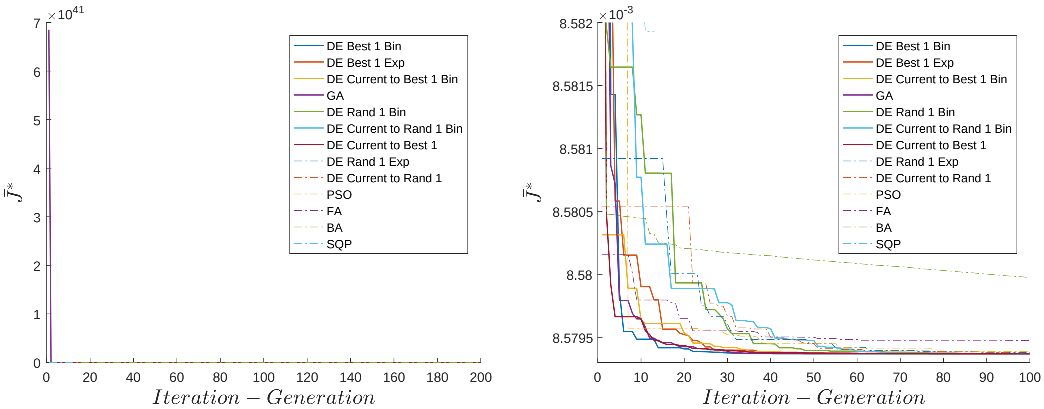
$Algorithm$	$Mean(\bar{J})$	$\sigma(\bar{J})$	$Mean(\bar{J}_{worst})$	$Mean(\bar{J}_{best})$	$Var(\bar{J})\%$	$\%BI$
DE-R1B	$8.5793692185e-3$	$8.9e-13$	$8.579369221e-3$	$8.57936921701e-3$	$8.8e-9$	30
DE-R1E	$8.5793694598e-3$	$2.8e-10$	$8.579371416e-3$	$8.57937110014e-3$	$5.3e-7$	30
DE-B1B	$\mathbf{8.5793692162e-3}$	$\mathbf{3.1e-17}$	$\mathbf{8.579369216e-3}$	$\mathbf{8.579369216251e-3}$	**0**	**30**
DE-B1E	$\mathbf{8.5793692163e-3}$	$\mathbf{2.6e-13}$	$\mathbf{8.579369217e-3}$	$\mathbf{8.579369216257e-3}$	$\mathbf{7.2e-11}$	**30**
DE-CR1	$8.5795668517e-3$	$3.4e-8$	$8.579644146e-3$	$8.57948993394e-3$	$1.4e-3$	0
DE-CB1	$8.5795131759e-3$	$2.0e-8$	$8.579556045e-3$	$8.57946539341e-3$	$1.1e-3$	11
DE-CR1B	$8.5794242931e-3$	$1.7e-8$	$8.579457351e-3$	$8.57939143686e-3$	$2.5e-4$	22
DE-CB1B	$8.5794288424e-3$	$1.3e-8$	$8.5794651230-3$	$8.57940095901e-3$	$3.6e-4$	22
PSO	$6.6666666666e+76$	$4.7e+77$	$3.333333333e+78$	$8.57962281939e-3$	$2.9e-3$	0
BA	$2.2e+78$	$7.6e+78$	$3.0e+79$	$8.58132029268e-3$	$2.2e-2$	0
FA	$6.6666666666e+76$	$4.7e+77$	$3.333333333e+78$	$8.57973314153e-3$	$4.2e-3$	0
GA	$8.5793692171e-3$	$9.4e-15$	$8.579369217e-3$	$8.57936921717e-3$	$1.0e-8$	30
SQP	$2.9255373256e+40$	$5.8e+40$	$1.170214930e+41$	$8.58192995713e-3$	100	0

marizes the number of wins of algorithms with such a test. The statistical significance is set as 5% and 10% to draw more reliable conclusions. Only in the particular case of SQP, the test is not carried out because of the lack of samples (only four runs from one hundred converge to a solution). We can conclude with a 99.99% confidence, that the best algorithm performance in control tuning for the Omnidirectional Mobile Robot is DE Best 1 Bin. When DE Best 1 Bin is compared with DE Best 1 Exp or GA, the comparison is not conclusive. In all other comparisons, the DE Best 1 Bin is superior. Then, the next best algorithms are given by DE Best 1 Exp and GA because they outperform the performance of seven and six algorithms, respectively and also, there are no other algorithms that outperform their behavior. As the main feature of the best algorithm (DE Best 1 Bin) and the second best (DE Best 1 Exp) is the inclusion of the best individual in the optimization process. This confirms that including such individual into the mutation process promotes the exploitation of the feasible region and hence enhances the ability to find better solutions. Furthermore, the worst performance is provided by the BA, FA and PSO algorithms due to they do not exceed the performance of any other algorithm. This bad behavior is attributed to the lack of a selection mechanism in those swarm optimization techniques. On the other hand, twenty-five comparisons cannot be conclusive (when $p - value > 0.1$ in Table S2, those comparative cases are named as Non Conclusive Data (NCD)) because there is no compelling evidence between samples to confirm which algorithm is better, indicating that the differences between such algorithms are due to chance.

The performance function convergence behavior through generations/iterations of the best run of each algorithm is displayed in Fig. 2a and zoom is shown in Fig. 2b. It is observed that SQP does not perform as well as the meta-heuristic algorithms because of the nonlinear characteristic of the optimization problem. On the other hand, if we set a convergence point of $5e - 9$ in the performance function value, the meta-heuristic algorithms which present faster convergence towards this point, are in the following order: DE Best 1 Bin and GA in the generation 94, DE Best 1 Exp in the generation 112, DE Current to Best 1 Bin in the generation 141 and DE Rand 1 Bin in the generation 199. Hence, DE Best 1 Bin, GA and DE Best 1 Exp provide the fastest convergence to the optimal solution. The above indicates that a balance in the convergence behavior at the beginning and in the last generations of the algorithm must be considered to find the best solution, i.e., promoting diversity of solutions at the beginning of the search, and the exploitation of the neighborhood solutions in the

Table 3. Number of wins of each algorithm based on the results of the multiple comparison test with different significance levels (5% and 10%). In addition Non Conclusive Data (NCD) are presented.

Algorithm	*Wins*(5%)	*Wins*(10%)	*NCD*
DE Best 1 Bin	**9**	**9**	2
DE Best 1 Exp	7	7	4
GA	6	7	4
DE Rand 1 Bin	5	5	5
DE Current to Best 1 Bin	5	5	5
DE Rand 1 Exp	3	3	5
DE Current to Rand 1 Bin	3	3	5
DE Current to Best 1	2	2	4
BA	0	0	3
PSO	0	0	3
FA	0	0	4
DE Current to Rand 1	0	0	6



a) Without zoom b) With zoom

Figure 2. Performance function behavior of the best runs of each algorithm.

last generations, can improve the performance of the algorithm. This fact is observed in the cases of DE Best 1 Bin, DE Best 1 Exp and GA, where the diversity of solutions is efficiently given in early generations, and the exploitation is exhaustively provided in the next generations.

4.2. *Optimal control gain analysis in a laboratory prototype*

After the analysis of the algorithm performance in simulation, we are also interested in performing an experimental study to know whether the best optimal design variable vectors obtained from each algorithm present different behavior for a real application tested in a laboratory prototype. This analysis is essential due to several optimal design variable vectors have a similar value as is displayed in Table 4. It is clear that if the optimization problem includes all uncertainties presented in the laboratory prototype, then there will not be significant differences in the performance of the control system for the mobile robot tracking. Nevertheless, several uncertainties are presented in the laboratory prototype which are not modeled or estimated in the optimization problem, such as the wear, the heating, the exact mobile robot parameters, etc. Then, in the next analysis, thirty-five independent real-time experiments through a laboratory prototype are carried out by each set of control gains shown in Table 4, to analyze and draw conclusions about the influence of such control gains (with small differences) in the control system performance within a real environment.

Table 4. The best optimal control gain vectors obtained from proposed algorithms with the corresponding performance function $\bar{J}$ in simulation. DE-R1B: DE Rand 1 Bin, DE-R1E: DE Rand 1 Bin, DE-B1B: DE Best 1 Bin, DE-B1E: DE Best 1 Exp, DE-CR1: DE Current to Rand 1, DE-CB1: Current to Best 1, DE-CR1B: Current to Rand 1 Bin, DE-CB1B: Current to Best 1 Bin.

Algorithm	kp_1^*	kp_2^*	kp_3^*	kd_1^*	kd_2^*	kd_3^*	$\bar{J}$ (SIMULATION)
DE − B1B	**1829.972**	**4937.868**	**2578.833**	**24.999**	**24.999**	**15.898**	**8.5793692162515e − 3**
DE − B1E	**1829.970**	**4937.871**	**2579.796**	**24.999**	**24.999**	**15.906**	**8.5793692162517e − 3**
DE-CB1B	1829.970	4937.858	2579.961	24.999	24.999	15.901	8.5793692162519$e-3$
GA	1830.083	4937.888	2579.399	25.000	24.999	15.904	8.5793692162631$e-3$
DE-R1B	1829.994	4937.759	2569.184	24.999	24.999	15.928	8.5793692164067$e-3$
DE-CR1B	1830.186	4937.781	2572.355	24.999	24.999	15.681	8.5793692167287$e-3$
DE-CB1	1829.801	4938.097	2408.343	24.999	24.999	14.798	8.5793692171431$e-3$
DE-R1E	1831.292	4937.167	2295.941	24.999	24.999	15.881	8.5793692226228$e-3$
DE-CR1	1809.147	4898.050	761.920	24.999	24.963	12.539	8.5793711757111$e-3$
PSO	1819.113	4933.728	2093.248	24.923	24.994	18.419	8.5793833630071$e-3$
FA	1810.534	3754.527	1227.129	24.999	21.668	9.820	8.5794742810212$e-3$
BA	1744.975	2520.802	233.882	24.828	19.555	4.090	8.5797370229618$e-3$
SQP	2728.778	804.134	900.835	25.000	17.212	13.134	8.5819299571437$e-3$

Fig. 3 shows the laboratory prototype which consists of an Omnidirectional Mobile Robot, a power source, and a computer display. The OMR was designed and developed by using optimization techniques to maximize its ability by properly locating the omnidirectional wheels (Villarreal-Cervantes et al. 2012). The OMR includes a Mini-ITX GA-D425TUD motherboard with an Intel Atom D525 processor, an embedded data acquisition system "Sensoray 626" and DC motor drivers "Advanced Motion model 12A8". The motherboard computes the control law in a Simulink program with Real-Time Windows Target, interacts with sensors and drivers through the data acquisition system and uses an odometry system (Guerrero-Castellanos et al. 2014) to obtain the position and velocity of the mobile robot at 200 Hz. The power source provides the corresponding voltage to the motherboard and drivers of actuators, and the computer display is only connected to the motherboard to start the experiments and to get the results. A graphical view of the closed-loop system of the OMR with the Cartesian space PD control is shown in Fig. 4.

The descriptive statistical summary of the thirty executions of the task in the Omnidirectional Mobile Robot with the best-fixed control gains obtained by each algorithm is shown in Table 5.

In order to draw general conclusions about the performance of the control gains obtained through different algorithms, the 95%-confidence Friedman test shown in Table S3 was first applied which based on the returned p-values ($2.3511e{-}42$), indicates that significant differences are presented among meta-heuristic algorithm runs and also the best rank in the Friedman test is provided by the DE Best 1 Bin.

Then, the Friedman test for multiple comparisons with the Holm and Shaffer post-hoc error correction methods is presented in Table S4, where a summary of the number of wins and Non-Conclusive Data (NCD) of the control system performance based on the corresponding gains is shown in Table 6. NCD refers to such comparisons where $p - value > 0.1$ and hence, there is no compelling evidence between samples to confirm that the differences between such control gains are not due to chance. This test uses the two-sided alternative hypothesis with a statistical significance of 5% and 10%. Based on the descriptive and inferential statistical tests we can conclude the following:

- With a 99.99% confidence in nine comparisons, the best control gains in the laboratory test are provided by DE Best 1 Bin. Only in the comparisons with

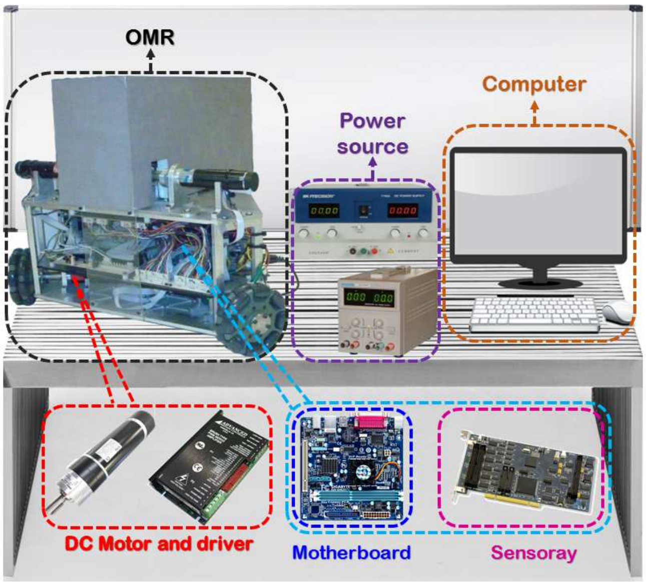


Figure 3. Omnidirectional mobile robot platform.

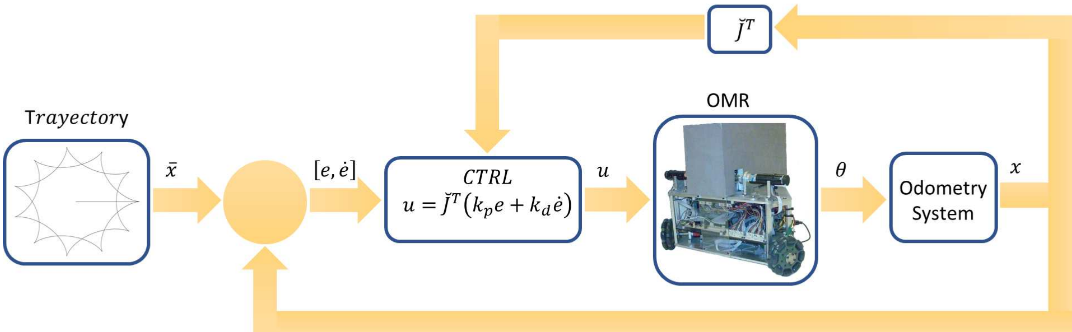


Figure 4. Block diagram of the control strategy.

SQP, BA, and FA the differences among samples are not conclusive.

- Enough statistical evidence indicates that control gains obtained by SQP, BA, and FA are the second, third and fourth best alternatives, respectively. This fact is attributed due to there being no statistical evidence that others algorithms outperform the trajectory tracking and the energy consumption of the control system in the Omnidirectional Mobile Robot.
- The results indicate that 48.71% of all comparisons cannot confirm the significant difference between the performances related to the proposed control gains (see the comparisons where $p - value > 0.1$ in Table S4 and $NCD \neq 0$ in Table 6), i.e., such control system performances can present performance differences due to chance. On the other hand, there is enough evidence that small variations in control gains can provide different performances in the tracking and energy consumption of the OMR. Two examples of this situation are the comparison between DE Current to Best 1 Bin with DE Rand 1 Bin, and DE Best 1 Bin with DE Best 1 Exp. In spite of presenting very similar control gains (see Table 4), the statistical test indicates that DE Current to Best 1 Bin is better than DE Rand 1 Bin, and DE Best 1 Bin is better than DE Best 1 Exp with a 99.99% confidence (see Table S4). For a particular case, the laboratory test

analysis statistically indicates that 75% of comparisons with the best control gains obtained by DE Best 1 Bin, present different performance functions in spite of presenting slightly different control gains. Hence, small differences in the mobile robot control gains in a real environment (see Table 4) can provide different control system performances.

- The standard deviation in the fourth column of Table 5 can provide quantitative evidence of the performance function variation due to environmental uncertainties. It is observed that control gains obtained by PSO present more sensitivity to the environment uncertainties.

Table 5. Descriptive statistical summary of the experimental results.

Algorithm	$Mean(\bar{J}^*)$	$Median(\bar{J}^*)$	$\sigma(\bar{J}^*)$	$\bar{J}^*_{worse}$	$\bar{J}^*_{best}$
DE Rand 1 Bin	58.1411	58.1658	0.3263	58.8191	57.6337
DE Rand 1 Exp	57.5457	57.5568	0.2257	58.1335	57.0373
DE Best 1 Bin	**56.8823**	**56.7379**	**0.3164**	**57.7021**	**56.5751**
DE Best 1 Exp	57.3113	57.4141	0.3245	57.8261	56.6051
DE Current to Rand 1	57.3202	57.3740	0.2583	57.8857	56.8000
DE Current to Best 1	57.9077	57.9367	0.2922	58.4882	57.4576
DE Current to Rand 1 Bin	57.9106	58.0015	0.3325	58.2249	56.9457
DE Current to Best 1 Bin	57.4090	57.3750	0.1634	57.7101	57.1545
PSO	57.4765	57.6654	0.4432	58.0740	56.7077
BA	57.0859	56.9366	0.3383	57.8345	56.7997
FA	57.1683	57.0377	0.2377	57.7247	56.9210
GA	57.3448	57.3297	0.1635	57.7916	57.0469
SQP	57.1372	57.2118	0.2359	57.4797	56.7439

Table 6. Number of wins of each algorithm based on the results of the multiple comparison test (laboratory test comparison) with different significance levels (5% and 10%) and Non Conclusive Data (NCD) of each control gain obtained by each algorithm.

Algorithm	$Wins(5\%)$	$Wins(10\%)$	NCD
DE Best 1 Bin	**9**	**9**	3
SQP	6	6	6
BA	5	5	7
FA	5	5	7
DE Current to Best 1 Bin	3	3	7
DE Current to Rand 1	3	3	8
DE Best 1 Exp	3	3	8
GA	3	3	8
PSO	2	2	6
DE Rand 1 Exp	1	1	7
DE Rand 1 Bin	0	0	2
DE Current to Rand 1 Bin	0	0	3
DE Current to Best 1	0	0	4

In Table 7, the Euclidean d_{xy} and angular d_ϕ distances between the generated real trajectory and the desired one, and also the energy consumption are displayed for the more reliable control gains in the experimental results. It is observed that the Cartesian distance error interval is suitable for the application and also similar energy is consumed. In Fig. 5, the OMR movements with the corresponding control input with the best gains obtained by DE Best 1 Bin is shown in experimentation and a video demonstration is given in https://youtu.be/Uishmg4mUBA. The control system can efficiently track the desired trajectory with a mean error of around $6.4261e-3$m and $1.0003e-3$rad; and also with an average energy consumption of $0.22942Wh$.

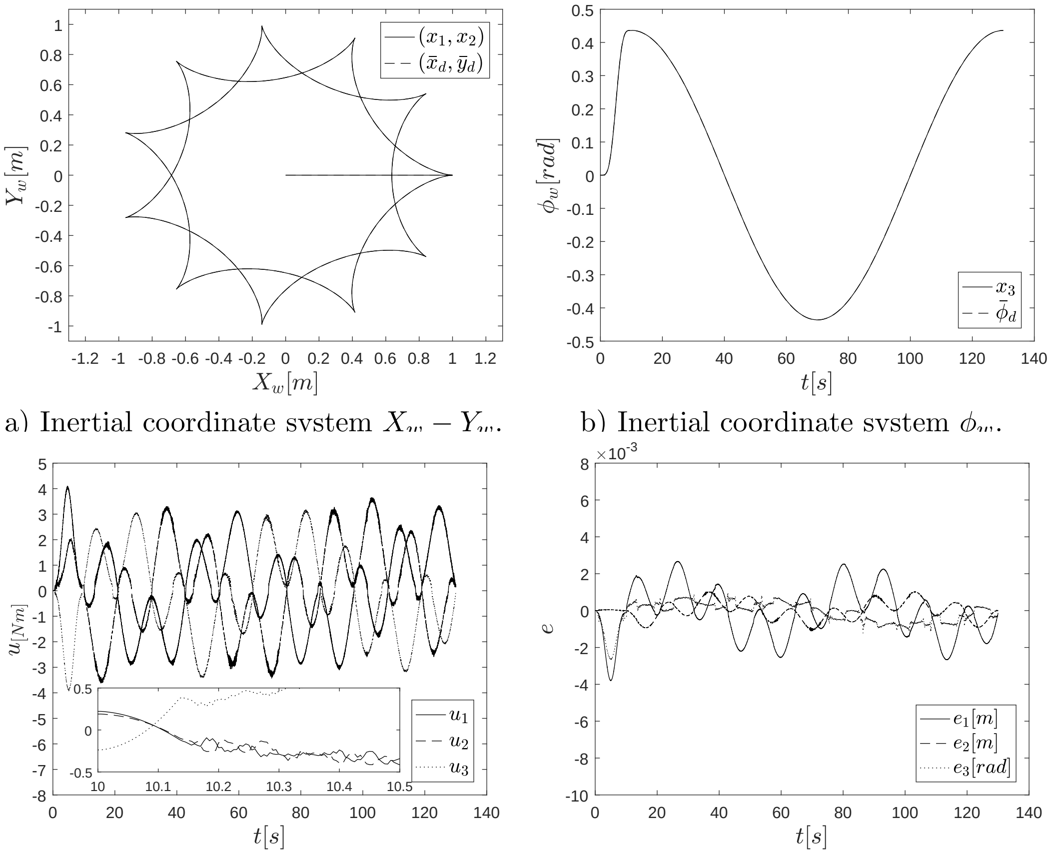
It is clear that the control tuning approach presented in Section 2 is based on a spe-

Table 7. Characteristic of the distance between the generated real trajectory and the desired one in the experimental results. *mean*, $[max, min]$ and E indicate the average, maximum and minimum of the distance vector d_{xy} and energy.

Algorithm	$mean(d_{xy})$ $[m]$	$[max, min]$ $[m]$	$mean(d_{\phi})$ $[rad]$	$[max, min]$ $[rad]$	E $[Wh]$
DE Best 1 Bin	$6.4261e-3$	$[1.3e-2,\ 3.4e-7]$	$1.0003e-3$	$[5.5e-3,\ 1.7e-7]$	0.2294
BA	$1.6003e-3$	$[3.9e-3,\ 9.5e-7]$	$5.1501e-3$	$[2.8e-2,\ 1.5e-7]$	0.2296
FA	$1.3892e-3$	$[3.8e-3,\ 2.2e-6]$	$9.8485e-4$	$[5.4e-3,\ 4.2e-8]$	0.2299
SQP	$2.7373e-3$	$[6.3e-3,\ 4.8e-6]$	$1.3396e-3$	$[7.4e-3,\ 2.0e-9]$	0.2300
LQR	$8.4354e-3$	$[1.8e-2,\ 4.7e-5]$	$6.4580e-4$	$[3.5e-3,\ 2.4e-8]$	0.2307

cific trajectory. Nevertheless, the best ("optimal") gains obtained with this approach can track other elemental paths that can be used for characterization in trajectory planning (Wang and Tsai 2004; Lacevic, Velagic, and Hebibovic 2005): the circle, the Lemniscate curve, and the straight line. The above is confirmed with the suitable performance of the proposal shown in Fig. 6, where different paths in $X_w - Y_w$ are tracked by the OMR using the control gains obtained with DE Best 1 Bin. Then, offline control tuning approach not only carries out a specific trajectory but also other trajectories with a suitable control performance.

As a matter of reference, a gain-scheduled Linear Quadratic Regulator (LQR) with feedforward (Åström and Murray 2008) is implemented in the experimental stage for the tracking of the hypocycloid path with the OMR. In this, a model linearization is required to find a set of state feedback gains for each sampling time instant. The regulator gains are offline scheduled, and each set of gains are obtained by using the LQR command in Matlab considering $Q = diag\{1e3, 1e3, 1e3, 1, 1, 1\} \in R^{6\times 6}$ and $R = diag\{0.01, 0.01, 0.01\} \in R^{3\times 3}$ into the single-objective optimization problem which uses a quadratic cost function related to the tracking error and the magnitude of the control signal. Table 7 indicates that the gain-scheduled LQR can also achieve promising results when compared with the optimally tuned PD controllers. The above because, like the proposal, the LQR is based on an optimization approach. Nevertheless, as the LQR approach is based on a model linearization (lack of nonlinear model dynamics into the optimization problem), the obtained set of LQR gains implemented in the real prototype present more Euclidian distance error and energy consumption than the control performance with the obtained gains based on meta-heuristic algorithms. On the other hand, the performance of the LQR is also contrasted with the huge complexity of the gain scheduling task. The above can be observed with the $26,000$ optimization processes required to schedule the gains for the tracking of the hypocycloid path, considering the sampling time $\triangle t = 5ms$ and the execution time $t_f = 130s$. Unlike the gain-scheduled LQR, the proposal requires a single optimization process to compute the optimal gains of the PD controller, which can be used to track not only the highly non-linear hypocycloid path but a set of elemental trajectories. Additionally, Fig. S1 shows the behavior of the LQR in the trajectory tracking task of the OMR. It is observed that the LQR tracks the hypocycloid path in the inertial coordinate system with a suitable orientation. Regarding the behavior of the PD controller tuned by DE Best 1 Bin (see Fig. 5), it can be noticed that the magnitude of Cartesian errors obtained with the LQR is considerably larger, while the magnitude of the orientation error in both cases is very similar.



a) Inertial coordinate system $X_w - Y_w$. b) Inertial coordinate system ϕ_w.

c) Control input. d) Tracking error.

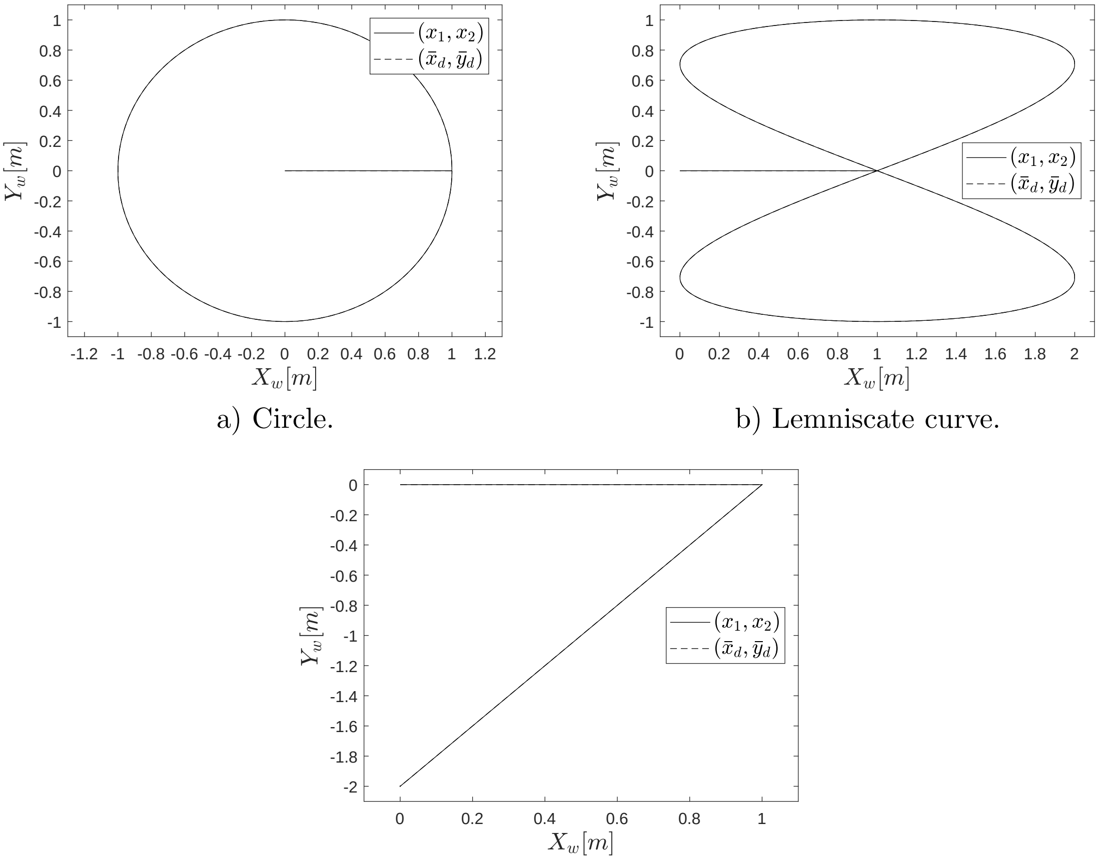
Figure 5. Experimental results of the trajectory tracking and control system behavior of the OMR using the gains obtained by DE Best 1 Bin.

5. Conclusion

In this paper, the Omnidirectional Mobile Robot control tuning is established in the framework of the optimization method where an offline dynamic optimization approach is proposed. The obtained solutions through a set of twelve different meta-heuristic algorithms and one gradient technique reduce the trajectory tracking error and the energy consumption of the control system. With the best control gains implemented in a real OMR prototype, the mean trajectory tracking and the angular position error are around $6.4261e-3$m and $1.0003e-3$rad, respectively, with an average energy consumption of $0.22942Wh$.

Statistical comparative analysis of algorithms indicates that the best algorithm for the control tuning for the OMR is given by DE Best 1 Bin because it includes the best individual in the evolutionary process which efficiently exploits the search space. In the OMR control tuning, the swarm based algorithms studied in this paper do not present an efficient search for solutions. This last is attributed to the lack of a replacement mechanism (selection process).

The OMR control tuning complexity is highlighted when the gradient-based algorithm is used. The main issue is the algorithm divergence (96% of runs) and the high sensitivity to viable initial conditions (4% of runs) such that the convergence to a local minimum is guaranteed. Therefore, this indicates that the optimization problem



a) Circle.

b) Lemniscate curve.

c) Straight line.

Figure 6. Experimental results with different trajectories developed by the mobile robot using the gains obtained by DE Best 1 Bin.

is highly multi-modal.

The experimental analysis indicates that in spite of presenting uncertainties not included in the optimization problem, small differences in the obtained control gains influence the OMR performance in the trajectory tracking and the energy consumption. Also, the suitable behavior in the control performance for simulation and experimental tests confirms that in spite of presenting uncertainties (inaccuracy in the kinematic and dynamic parameters and the lack of nonlinear dynamics such as the motor dynamics) in the experimental tests, the obtained control gains in simulation can be used in the real scenario.

Even when the OMR control is tuned to track a highly non-linear trajectory (hypocycloid path), the obtained control parameters can be used to follow other elemental curves with an outstanding performance i.e., the proposal presents a higher trajectory change flexibility.

Comparisons with a gain-scheduled LQR based on a model linearization highlight advantages of optimization tuning methods and particularly some benefits of the proposal including a higher performance in the trajectory tracking task of the OMR and a less computational complexity.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Supplementary material

"Meta-heuristic algorithms for the control tuning of omnidirectional mobile robots"

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Engineering Optimization

Table S1. Ranks achieved by the Friedman test in the main study case. The computed statistics and the related p-value are also shown.

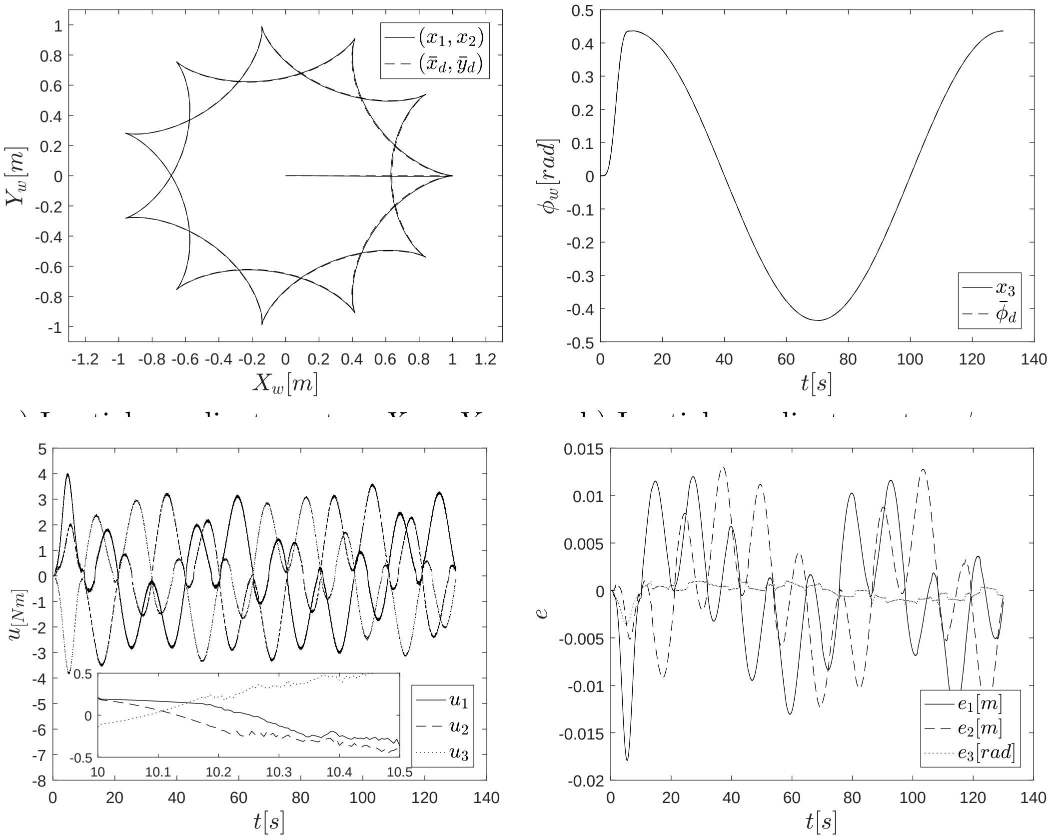
Algorithms	Ranks
DE Best 1 Bin	1
DE Best 1 Exp	2.367
GA	3.667
DE Current to Best 1 Bin	4.2
DE Rand 1 Bin	4.433
DE Rand 1 Exp	6.433
DE Current to Rand 1 Bin	6.6
DE Current to Best 1	7.7
DE Current to Rand 1	9
FA	10.2
BA	11.12
PSO	11.28
Statistic	301.2
p-value	4.8568E-58

Table S2. Adjusted p-values for multiple comparison test among all methods. Boldface indicates the 5% significance level. Boldface with asterisks indicates the 10% significance level.

Hypotesis	Unajusted p	Holm	Shaffer	z
BA vs **DE Best 1 Bin**	**0**	**0**	**0**	10.87
BA vs **DE Best 1 Exp**	**0**	**0**	**0**	9.399
BA vs **DE Current to Best 1**	**2.4247E-04**	**7.2740E-03**	**7.2740E-03**	3.67
BA vs **DE Current to Best 1 Bin**	**1.0880E-13**	**6.0929E-12**	**5.9841E-12**	7.43
BA vs DE Current to Rand 1	**2.2986E-02**	3.9076E-01	3.9076E-01	2.274
BA vs **DE Current to Rand 1 Bin**	**1.2242E-06**	**4.7744E-05**	**4.7744E-05**	4.852
BA vs **DE Rand 1 Bin**	**7.0188E-13**	**3.7200E-11**	**3.2287E-11**	7.179
BA vs **DE Rand 1 Exp**	**4.8867E-07**	**2.0524E-05**	**1.9058E-05**	5.031
BA vs FA	3.2479E-01	1	1	0.9847
BA vs **GA**	**1.1102E-15**	**6.4393E-14**	**6.1062E-14**	8.003
BA vs PSO	8.5792E-01	1	1	-0.179
DE Best 1 Bin vs DE Best 1 Exp	1.4209E-01	1	1	-1.468
DE Best 1 Bin vs DE Current to Best 1	**6.1573E-13**	**3.3249E-11**	**2.8324E-11**	-7.197
DE Best 1 Bin vs DE Current to Best 1 Bin	**5.8743E-04**	**1.6448E-02**	**1.6448E-02**	-3.437
DE Best 1 Bin vs DE Current to Rand 1	**0**	**0**	**0**	-8.593
DE Best 1 Bin vs DE Current to Rand 1 Bin	**1.7948E-09**	**8.6151E-08**	**8.2562E-08**	-6.015
DE Best 1 Bin vs DE Rand 1 Bin	**2.2603E-04**	**7.0070E-03**	**7.0070E-03**	-3.688
DE Best 1 Bin vs DE Rand 1 Exp	**5.3361E-09**	**2.5080E-07**	**2.4546E-07**	-5.836
DE Best 1 Bin vs FA	**0**	**0**	**0**	-9.882
DE Best 1 Bin vs GA	**4.1772E-03**	1.0443E-01	1.0443E-01	-2.864
DE Best 1 Bin vs PSO	**0**	**0**	**0**	-11.05
DE Best 1 Exp vs DE Current to Best 1	**1.0107E-08**	**4.6493E-07**	**4.6493E-07**	-5.729
DE Best 1 Exp vs DE Current to Best 1 Bin	**4.8917E-02**	6.8484E-01	6.8484E-01	-1.969
DE Best 1 Exp vs DE Current to Rand 1	**1.0383E-12**	**5.3991E-11**	**4.7761E-11**	-7.125
DE Best 1 Exp vs DE Current to Rand 1 Bin	**5.4331E-06**	**2.0646E-04**	**2.0102E-04**	-4.547
DE Best 1 Exp vs DE Rand 1 Bin	**2.6422E-02**	4.2275E-01	4.2275E-01	-2.22
DE Best 1 Exp vs DE Rand 1 Exp	**1.2522E-05**	**4.6330E-04**	**4.6330E-04**	-4.368
DE Best 1 Exp vs FA	**0**	**0**	**0**	-8.414
DE Best 1 Exp vs GA	1.6259E-01	1	1	-1.396
DE Best 1 Exp vs PSO	**0**	**0**	**0**	-9.578
DE Current to Best 1 vs **DE Current to Best 1 Bin**	**1.7018E-04**	**5.4459E-03**	**5.2757E-03**	3.76
DE Current to Best 1 vs DE Current to Rand 1	1.6259E-01	1	1	-1.396
DE Current to Best 1 vs DE Current to Rand 1 Bin	2.3737E-01	1	1	1.182
DE Current to Best 1 vs **DE Rand 1 Bin**	**4.4986E-04**	**1.3046E-02**	**1.3046E-02**	3.509
DE Current to Best 1 vs DE Rand 1 Exp	1.7363E-01	1	1	1.361
DE Current to Best 1 vs FA	**7.2436E-03**	1.6660E-01	1.6660E-01	-2.685
DE Current to Best 1 vs **GA**	**1.4743E-05**	**5.3074E-04**	**5.3074E-04**	4.332
DE Current to Best 1 vs PSO	**1.1854E-04**	**3.9120E-03**	**3.7459E-03**	-3.849
DE Current to Best 1 Bin vs DE Current to Rand 1	**2.5224E-07**	**1.0847E-05**	**9.8375E-06**	-5.156
DE Current to Best 1 Bin vs DE Current to Rand 1 Bin	**9.9370E-03**	2.1861E-01	2.1861E-01	-2.578
DE Current to Best 1 Bin vs DE Rand 1 Bin	8.0209E-01	1	1	-0.2506
DE Current to Best 1 Bin vs DE Rand 1 Exp	**1.6441E-02**	3.1237E-01	3.1237E-01	-2.399
DE Current to Best 1 Bin vs FA	**1.1557E-10**	**5.7787E-09**	**5.3164E-09**	-6.445
DE Current to Best 1 Bin vs GA	5.6672E-01	1	1	0.5729
DE Current to Best 1 Bin vs PSO	**2.7756E-14**	**1.5821E-12**	**1.5266E-12**	-7.609
DE Current to Rand 1 vs DE Current to Rand 1 Bin	**9.9370E-03**	2.1861E-01	2.1861E-01	2.578
DE Current to Rand 1 vs **DE Rand 1 Bin**	**9.3243E-07**	**3.7297E-05**	**3.6365E-05**	4.905
DE Current to Rand 1 vs DE Rand 1 Exp	**5.8327E-03**	1.3998E-01	1.3998E-01	2.757
DE Current to Rand 1 vs FA	1.9740E-01	1	1	-1.289
DE Current to Rand 1 vs **GA**	**1.0107E-08**	**4.6493E-07**	**4.6493E-07**	5.729
DE Current to Rand 1 vs PSO	**1.4179E-02**	2.8358E-01	2.8358E-01	-2.453
DE Current to Rand 1 Bin vs DE Rand 1 Bin	**1.9945E-02**	3.5902E-01	3.5902E-01	2.327
DE Current to Rand 1 Bin vs DE Rand 1 Exp	8.5792E-01	1	1	0.179
DE Current to Rand 1 Bin vs FA	**1.1017E-04**	**3.7459E-03**	**3.7459E-03**	-3.867
DE Current to Rand 1 Bin vs **GA**	**1.6277E-03**	**4.3947E-02**	**4.3947E-02**	3.151
DE Current to Rand 1 Bin vs PSO	**4.8867E-07**	**2.0524E-05**	**1.9058E-05**	-5.031
DE Rand 1 Bin vs DE Rand 1 Exp	**3.1686E-02**	4.7530E-01	4.7530E-01	-2.148
DE Rand 1 Bin vs FA	**5.8510E-10**	**2.8670E-08**	**2.6915E-08**	-6.194
DE Rand 1 Bin vs GA	4.1021E-01	1	1	0.8235
DE Rand 1 Bin vs PSO	**1.8652E-13**	**1.0258E-11**	**1.0258E-11**	-7.358
DE Rand 1 Exp vs FA	**5.2089E-05**	**1.8231E-03**	**1.7710E-03**	-4.046
DE Rand 1 Exp vs **GA** **	**2.9599E-03**	**7.6956E-02** **	**7.3996E-02** **	2.972
DE Rand 1 Exp vs PSO	**1.8911E-07**	**8.3208E-06**	**7.3753E-06**	-5.21
FA vs **GA**	**2.2518E-12**	**1.1484E-10**	**1.0358E-10**	7.018
FA vs PSO	2.4455E-01	1	1	-1.164
GA vs PSO	**2.2204E-16**	**1.3101E-14**	**1.2212E-14**	-8.182

Table S3. Ranks achieved by the Friedman test in the main study case (laboratory test comparison). The computed statistics and the related p-value are also shown.

Algorithms	Ranks
DE Best 1 Bin	2.2
SQP	3.533
BA	3.967
FA	4.333
DE Best 1 Exp	5.933
GA	6.2
DE Current to Rand 1	6.5
DE Current to Best 1 Bin	6.9
PSO	8.033
DE Rand 1 Exp	8.733
DE Current to Best 1	11
DE Current to Rand 1 Bin	11.3
DE Rand 1 Bin	12.37
Statistic	238.9
p-value	2.8262E-44



c) Control input.

d) Tracking error.

Figure S1. Experimental results of the trajectory tracking and control system behavior of the OMR using the gains obtained by the LQR.

Table S4. Adjusted p-values for the multiple comparison test among all methods (laboratory test comparison). Boldface indicates the 5% significance level. Boldface with asterisks indicates the 10% significance level.

Hypotesis	Unajusted p		Holm	Shaffer	z
BA vs DE Best 1 Bin	**7.8929E-02**	**	1	1	1.757
BA vs DE Best 1 Exp	**5.0485E-02**	**	1	1	-1.956
BA vs DE Current to Best 1	**2.6605E-12**		**1.8358E-10**	**1.7560E-10**	-6.995
BA vs DE Current to Best 1 Bin	**3.5322E-03**		1.2716E-01	1.2716E-01	-2.917
BA vs DE Current to Rand 1	**1.1756E-02**		3.6445E-01	3.6445E-01	-2.519
BA vs DE Current to Rand 1 Bin	**3.0331E-13**		**2.1232E-11**	**2.0019E-11**	-7.293
BA vs DE Rand 1 Bin	**0**		**0**	**0**	-8.354
BA vs DE Rand 1 Exp	**2.1329E-06**		**1.1518E-04**	**1.0238E-04**	-4.74
BA vs FA	7.1538E-01		1	1	-0.3646
BA vs GA	**2.6349E-02**		7.1142E-01	7.1142E-01	-2.221
BA vs PSO	**5.2489E-05**		**2.3620E-03**	**2.3620E-03**	-4.044
BA vs SQP	6.6651E-01		1	1	0.4309
DE Best 1 Bin vs DE Best 1 Exp	**2.0501E-04**		**8.8154E-03**	**8.6103E-03**	-3.713
DE Best 1 Bin vs DE Current to Best 1	**0**		**0**	**0**	-8.752
DE Best 1 Bin vs DE Current to Best 1 Bin	**2.9524E-06**		**1.5648E-04**	**1.4171E-04**	-4.674
DE Best 1 Bin vs DE Current to Rand 1	**1.9002E-05**		**8.9308E-04**	**8.7408E-04**	-4.276
DE Best 1 Bin vs DE Current to Rand 1 Bin	**0**		**0**	**0**	-9.05
DE Best 1 Bin vs DE Rand 1 Bin	**0**		**0**	**0**	-10.11
DE Best 1 Bin vs DE Rand 1 Exp	**8.1754E-11**		**5.3958E-09**	**5.3958E-09**	-6.497
DE Best 1 Bin vs FA	**3.3873E-02**		8.1295E-01	8.1295E-01	-2.122
DE Best 1 Bin vs GA	**6.9509E-05**		**3.0584E-03**	**2.9194E-03**	-3.978
DE Best 1 Bin vs PSO	**6.5845E-09**		**4.0824E-07**	**3.6873E-07**	-5.801
DE Best 1 Bin vs SQP	1.8484E-01		1	1	-1.326
DE Best 1 Exp vs DE Current to Best 1	**4.6858E-07**		**2.6709E-05**	**2.6240E-05**	-5.039
DE Best 1 Exp vs DE Current to Best 1 Bin	3.3638E-01		1	1	-0.9613
DE Best 1 Exp vs DE Current to Rand 1	5.7306E-01		1	1	-0.5635
DE Best 1 Exp vs DE Current to Rand 1 Bin	**9.4446E-08**		**5.6668E-06**	**5.2890E-06**	-5.337
DE Best 1 Exp vs DE Rand 1 Bin	**1.5754E-10**		**1.0240E-08**	**8.8223E-09**	-6.398
DE Best 1 Exp vs DE Rand 1 Exp	**5.3598E-03**		1.8759E-01	1.8223E-01	-2.785
DE Best 1 Exp vs FA	1.1157E-01		1	1	1.591
DE Best 1 Exp vs GA	7.9086E-01		1	1	-0.2652
DE Best 1 Exp vs PSO	**3.6759E-02**		8.4546E-01	8.4546E-01	-2.088
DE Best 1 Exp vs SQP	**1.6997E-02**		4.9291E-01	4.9291E-01	2.387
DE Current to Best 1 vs **DE Current to Best 1 Bin**	**4.5540E-05**		**2.0948E-03**	**2.0948E-03**	4.077
DE Current to Best 1 vs **DE Current to Rand 1**	**7.6338E-06**		**3.9696E-04**	**3.6642E-04**	4.475
DE Current to Best 1 vs DE Current to Rand 1 Bin	7.6544E-01		1	1	-0.2983
DE Current to Best 1 vs DE Rand 1 Bin	1.7410E-01		1	1	-1.359
DE Current to Best 1 vs DE Rand 1 Exp	**2.4185E-02**		6.7718E-01	6.7718E-01	2.254
DE Current to Best 1 vs **FA**	**3.3583E-11**		**2.2501E-09**	**2.2165E-09**	6.63
DE Current to Best 1 vs **GA**	**1.8100E-06**		**1.0136E-04**	**1.0136E-04**	4.774
DE Current to Best 1 vs PSO	**3.1744E-03**		1.2063E-01	1.2063E-01	2.95
DE Current to Best 1 vs **SQP**	**1.1235E-13**		**7.9772E-12**	**7.4154E-12**	7.426
DE Current to Best 1 Bin vs DE Current to Rand 1	6.9078E-01		1	1	0.3978
DE Current to Best 1 Bin vs DE Current to Rand 1 Bin	**1.2101E-05**		**6.0506E-04**	**5.8086E-04**	-4.376
DE Current to Best 1 Bin vs DE Rand 1 Bin	**5.4323E-08**		**3.3137E-06**	**3.0421E-06**	-5.437
DE Current to Best 1 Bin vs DE Rand 1 Exp	**6.8268E-02**	**	1	1	-1.823
DE Current to Best 1 Bin vs FA	**1.0695E-02**		3.5292E-01	3.4222E-01	2.553
DE Current to Best 1 Bin vs GA	4.8634E-01		1	1	0.6961
DE Current to Best 1 Bin vs PSO	2.5970E-01		1	1	-1.127
DE Current to Best 1 Bin vs **SQP**	**8.1363E-04**		**3.2545E-02**	**3.1731E-02**	3.348
DE Current to Rand 1 vs DE Current to Rand 1 Bin	**1.8100E-06**		**1.0136E-04**	**1.0136E-04**	-4.774
DE Current to Rand 1 vs DE Rand 1 Bin	**5.4003E-09**		**3.4022E-07**	**3.0242E-07**	-5.834
DE Current to Rand 1 vs DE Rand 1 Exp	**2.6349E-02**		7.1142E-01	7.1142E-01	-2.221
DE Current to Rand 1 vs FA	**3.1183E-02**		7.7958E-01	7.7958E-01	2.155
DE Current to Rand 1 vs GA	7.6544E-01		1	1	0.2983
DE Current to Rand 1 vs PSO	1.2729E-01		1	1	-1.525
DE Current to Rand 1 vs SQP	**3.1744E-03**		1.2063E-01	1.2063E-01	2.95
DE Current to Rand 1 Bin vs DE Rand 1 Bin	2.8879E-01		1	1	-1.061
DE Current to Rand 1 Bin vs DE Rand 1 Exp	**1.0695E-02**		3.5292E-01	3.4222E-01	2.553
DE Current to Rand 1 Bin vs **FA**	**4.2597E-12**		**2.8966E-10**	**2.8114E-10**	6.928
DE Current to Rand 1 Bin vs **GA**	**3.9386E-07**		**2.2844E-05**	**2.2056E-05**	5.072
DE Current to Rand 1 Bin vs **PSO**	**1.1595E-03**		**4.5219E-02**	**4.5219E-02**	3.249
DE Current to Rand 1 Bin vs **SQP**	**1.1324E-14**		**8.1535E-13**	**7.4740E-13**	7.724
DE Rand 1 Bin vs **DE Rand 1 Exp**	**3.0231E-04**		**1.2395E-02**	**1.1790E-02**	3.613
DE Rand 1 Bin vs **FA**	**1.3323E-15**		**9.7256E-14**	**8.7930E-14**	7.989
DE Rand 1 Bin vs **GA**	**8.6405E-10**		**5.5299E-08**	**4.8387E-08**	6.133
DE Rand 1 Bin vs **PSO**	**1.6366E-05**		**7.8554E-04**	**7.8554E-04**	4.309
DE Rand 1 Bin vs **SQP**	**0**		**0**	**0**	8.785
DE Rand 1 Exp vs **FA**	**1.2101E-05**		**6.0506E-04**	**5.8086E-04**	4.376
DE Rand 1 Exp vs GA	**1.1756E-02**		3.6445E-01	3.6445E-01	2.519
DE Rand 1 Exp vs PSO	4.8634E-01		1	1	0.6961
DE Rand 1 Exp vs **SQP**	**2.3241E-07**		**1.3712E-05**	**1.3015E-05**	5.171
FA vs GA	**6.3399E-02**	**	1	1	-1.856
FA vs PSO	**2.3359E-04**		**9.8107E-03**	**9.8107E-03**	-3.68
FA vs SQP	4.2627E-01		1	1	0.7956
GA vs PSO	**6.8268E-02**	**	1	1	-1.823
GA vs SQP	**8.0023E-03**		2.7208E-01	2.7208E-01	2.652
PSO vs **SQP**	**7.6338E-06**		**3.9696E-04**	**3.6642E-04**	4.475