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# DYNAMIC APPROACH TO OPTIMUM SYNTHESIS OF A FOUR-BAR MECHANISM USING A SWARM INTELLIGENCE ALGORITHM

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This paper presents a dynamic approach to the synthesis of a crank-rocker four-bar mechanism which provide an efficient and accurate mechanism synthesis tools for the symmetric motion of the rocker. A swarm intelligence algorithm called artificial bee colony (ABC) algorithm is used. Nevertheless, some modifications in the ABC algorithm are carried out to accelerate the search in the unfeasible space without compromising the explorative capabilities.

*Keywords:* ABC algorithm, four-bar mechanism, synthesis

*Classification:* 93E12, 62A10

## 1. INTRODUCTION

Mechanism are used continuously in a variety of machines and electromechanical devices. Several working processes require a continuous input motion that provide a non-symmetrical or complex output motion. It is difficult to design a mechanism which achieves an adequate output motion as it is specified, so that the mechanism design requires the task to be performed. The synthesis of the mechanism could be achieved by using graphical, analytical and numerical methods [18], [7], [8]. The computational cost increase as the number of precision points increase [8]. The formulation of the mechanism synthesis as an optimization problem is an alternative approach to find the dimension of the links by using heuristic or gradient optimization techniques [9], [10].

In [11], the path synthesis of a four-bar mechanism (FBM) to track more than five points in the coupler link is solved by using three different evolutionary algorithms with a new refinement technique. In that work the DE shows faster convergence to the optimal result and a smaller error of adjustment to target points than the genetic algorithm (GA) and particle swarm optimization (PSO). The work presented in [12] proposed an evolutionary algorithm to solve the path synthesis problem of a four-bar linkage. In [13] another design approach of a four-bar mechanism for path generation purposes is formulated as a constrained multi-objective optimization problem. The tracking error, the transmission angle's deviation and the maximum angular velocity ratio are introduced as the mechanical performance index. It proposes a hybridization of the traditional NSGA-II algorithm with an adaptive local search mechanism which presents a superior mechanical design in terms of energy efficiency and practical viability.

On the other hand, there are researches which focus on finding better heuristic algorithms to tackle several complex problems of real world. Scientists have been observing the nature for years in order to improve their heuristic algorithms. Natural selection eliminates species with poor foraging behavior and favor species with high foraging behavior which is essential for maximization of species fitness. Hence, the real world optimization problems can be solved by using heuristic algorithms based on the natural selection called bio-inspired algorithms.

Many bio-inspired algorithms using the concept of swarm intelligence (SI) such as PSO [20], artifificial fish swarm algorithm [22], ant colony [21] and bacterial foraging algorithm [19]. They have been studied and used in several optimization problems. In the SI models the population of interacting agents or swarms are able to self-organize. In recent years, a new swarm intelligence , artificial bee colony algorithm [14], developed by Prof. Karaboga in 2005 has been used. The artificial bee colony (ABC) algorithm simulates the intelligent foraging behavior of honey bee swarms. The ABC algorithm has had a rapid growth and it has been used in machining processes [15], in the filter design [16] and in the chaos control and synchronization of nonlinear systems [17].

In this paper, the synthesis of a four-bar mechanism with spring and damping forces (FBM-SDF) to provide a symmetric motion in its rocker link is formulated as a dynamic optimization problem. The dynamic behavior of the mechanism is included into the optimization problem as a dynamic constraint. This constraint makes that the angular velocity of the actuator is not constant. Another constraints are presented to help in the identification of the feasible region of the solution space. The ABC algorithm is modified in order to fulfill the kinematic performance.

The rest of the paper is structured as follows: in Section 2 describes the dynamic model of the FBM-SDF mechanism and the driving motor. Section 3 establishes the design variable vector, the design objective and the constraints of the dynamic optimization problem. The dynamic approach of the mechanism synthesis is stated in Section 4. In section 5 the ABC algorithm is explained as well as its modification for accelerating the search in the unfeasible space. The discussion of the algorithm and the optimum design are then presented in Section 6. Finally the conclusion are drawn in Section 7.

## 2. MECHANISM SYNTHESIS PROBLEM

One of the most used mechanism in the industrial machinery is the four-bar mechanism (FBM). That is because the operational principle of this mechanism enables a coupling with a continuous rotational power supply in order to obtain a desired output motion.

In [5], an optimum synthesis of a FBM was carried out in order to meet the necessary input motion to continuously variable transmission. A mechanical synthesis of a FBM is established as an optimization problem. The kinematic analysis of the FBM is presented and objective functions as well as constraints are proposed. The solutions are obtained with a swarm intelligent algorithm. The goal of the mechanical design is to obtain a set of dimensions of the mechanical elements of the FBM, which allow a large amplitude on the motion of the rocker as well as to ensure a smooth transmission of force and speed on the joint of the connecting rod and the rocker of the FBM.

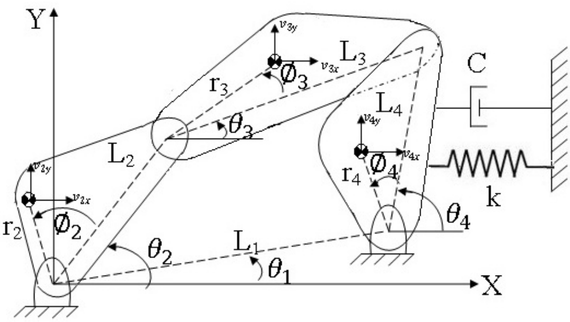



**Fig. 1.** A four-bar mechanism in a crank-rocker configuration.

### 2.1. Kinematic analysis of the FBM-SDF

The kinematic analysis of the four-bar mechanism with spring and damping forces (FBM-SDF) presents the same procedure as in the FBM. The kinematics of this mechanism has been widely described in [7]. The crank-rocker schematic representation is shown in Fig. 1. This FBM-SDF configuration is composed by a reference bar ($L_1$), a crank link ($L_2$), a coupler link ($L_3$) and a rocker link ($L_4$). Where $\theta_i$ with $i = 1, 2, 3, 4$, is the *i-th* angle between the horizontal axis and the *i-th* link in the counterclockwise direction. Also, the center of mass of each bar is denoted by a black-white circle and their location are described by $r_i$ and $\phi_1$ where $i = 2, 3, 4$.

From the kinematic analysis, the linear and angular velocity of the $i-th$ mass center of the links with respect to the inertial frame $X - Y$ are stated in (1)-(3).

$$\dot{\theta}_i = \gamma_i \dot{\theta}_2 \tag{1}$$

$$\upsilon_{ix} = \alpha_i \dot{\theta}_2 \tag{2}$$

$$\upsilon_{iy} = \beta_i \dot{\theta}_2 \tag{3}$$

where:

$$\alpha_2 = -r_2 \sin(\theta_2 + \phi_2) \tag{4}$$
$$\alpha_3 = -L_2 \sin\theta_2 - r_3\gamma_3 \sin(\theta_3 + \phi_3) \tag{5}$$
$$\alpha_4 = -r_4\gamma_4 \sin(\theta_4 + \phi_4) \tag{6}$$
$$\beta_2 = r_2 \cos(\theta_2 + \phi_2) \tag{7}$$
$$\beta_3 = L_2 \cos\theta_2 - r_3\gamma_3 \cos(\theta_3 + \phi_3) \tag{8}$$
$$\beta_4 = r_4\gamma_4 \cos(\theta_4 + \phi_4) \tag{9}$$
$$\gamma_2 = 1 \tag{10}$$
$$\gamma_3 = \frac{L_2 \sin(\theta_4 - \theta_2)}{L_3 \sin(\theta_3 - \theta_4)} \tag{11}$$
$$\gamma_4 = \frac{L_2 \sin(\theta_3 - \theta_2)}{L_3 \sin(\theta_3 - \theta_4)} \tag{12}$$

The closed-loop equation of the FBM-SDF is used to compute the passive angles $\theta_3$ and $\theta_4$ as it is shown in (13)-(14).

$$\theta_3 = 2arctan\left[\frac{-B_1 + \sqrt{B_1^2 + A_1^2 - C_1^2}}{C_1 - A_1}\right] \tag{13}$$

$$\theta_4 = 2arctan\left[\frac{-E_1 - \sqrt{D_1^2 + E_1^2 - F_1^2}}{F_1 - D_1}\right] \tag{14}$$

where:

$$A_1 = 2L_3\left(L_2cos\theta_2 - L_1cos\theta_1\right) \tag{15}$$
$$B_1 = 2L_3\left(L_2sin\theta_2 - L_1sin\theta_1\right) \tag{16}$$
$$C_1 = L_1^2 + L_2^2 + L_3^2 - L_4^2 - 2L_1L_2cos\left(\theta_1 - \theta_2\right) \tag{17}$$
$$D_1 = 2L_4\left(L_1cos\theta_1 - L_2cos\theta_2\right) \tag{18}$$
$$E_1 = 2L_4\left(L_1sin\theta_1 - L_2sin\theta_2\right) \tag{19}$$
$$F_1 = L_1^2 + L_2^2 + L_4^2 - L_3^2 - 2L_1L_2cos\left(\theta_1 - \theta_2\right) \tag{20}$$

### 2.2. Dynamic analysis of the FBM-SDF

The four-bar mechanism with spring and damping forces (FBM-SDF) has one degree of freedom ($dof$) in the crank (bar $L_2$). This $dof$ is actuated by a DC motor. From the schematic representation of the mechanism in Fig. 1, the mass, the inertia, the length, the mass center length and the mass center angle of the $i-th$ bar are represented by $m_i, J_i, L_i, r_i, \phi_i$, respectively. The stiffness constant of the spring and the damping coefficient of the damper are represented by $k$ and $C$.

Let $\theta_2$ and $\dot{\theta}_2$ the generalized coordinate and velocity of the FBM-SDF, the Lagrange's equation is formulated in (21). The variables $K$, $P$ and $D$ are the kinetic energy (22), potential energy (23) and the Rayleighs dissipation function (24), respectively. The angular position when the spring is in equilibrium is named as $\theta_{4,0}$.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) - \frac{\partial \mathcal{L}}{\partial \theta_2} + \frac{\partial D}{\partial \dot{\theta}_2} = T \tag{21}$$

where:

$$K = \sum_{i=2}^{4}\left(\frac{1}{2}m_i\left(\upsilon_{ix}^2 + \upsilon_{iy}^2\right) + \frac{1}{2}J_i\dot{\theta}_i^2\right) = \frac{1}{2}A\left(\theta_2\right)\dot{\theta}_2^2 \tag{22}$$

$$P = \frac{1}{2}k\left(\theta_4 - \theta_{4,0}\right)^2 \tag{23}$$

$$D = \frac{1}{2}C\dot{\theta}_4^{\ 2} \tag{24}$$

$$A\left(\theta_2\right) = \sum_{i=2}^{4}\left(m_i\left(\alpha_i^2 + \beta_i^2\right) + \gamma_i^2 J_i\right) \tag{25}$$

Computing the total and partial derivative of (21), the motion equation of the FBM-SDF results in (26).

$$T = A\left(\theta_2\right)\ddot{\theta}_2 + \frac{1}{2}\frac{dA\left(\theta_2\right)}{d\theta_2}\dot{\theta}_2^2 + k\gamma_4\left(\theta_4 - \theta_{4,0}\right) + C\gamma_4^2\dot{\theta}_2 \tag{26}$$

where:

$$A\left(\theta_2\right) = C_0 + C_1\gamma_3^2 + C_2\gamma_4^2 + C_3\gamma_3\cos\left(\theta_2 - \theta_3 - \phi_3\right) \tag{27}$$

$$\begin{aligned}\frac{dA\left(\theta_2\right)}{d\theta_2} &= 2C_1\gamma_3\frac{d\gamma_3}{d\theta_2} + 2C_2\gamma_4\frac{d\gamma_4}{d\theta_2}\\ &+ C_3\frac{d\gamma_3}{d\theta_2}\cos\left(\theta_2 - \theta_3 - \phi_3\right)\\ &- C_3\gamma_3\left(1 - \gamma_3\right)\sin\left(\theta_2 - \theta_3 - \phi_3\right)\end{aligned} \tag{28}$$

$$C_0 = J_2 + m_2r_2^2 + m_3L_2^2 \tag{29}$$

$$C_1 = J_3 + m_3r_3^2 \tag{30}$$

$$C_2 = J_4 + m_4r_4^2 \tag{31}$$

$$C_4 = 2m_3L_2r_3 \tag{32}$$

$$\frac{d\gamma_3}{d\theta_2} = \frac{L_2\left(D_1 + D_2\right)}{L_3\sin^2\left(\theta_3 - \theta_4\right)} \tag{33}$$

$$\frac{d\gamma_4}{d\theta_2} = \frac{L_2\left(D_3 + D_4\right)}{L_4\sin^2\left(\theta_3 - \theta_4\right)} \tag{34}$$

$$D_1 = \left(\gamma_4 - 1\right)\sin\left(\theta_3 - \theta_4\right)\cos\left(\theta_4 - \theta_2\right) \tag{35}$$

$$D_2 = \left(\gamma_4 - \gamma_3\right)\sin\left(\theta_4 - \theta_2\right)\cos\left(\theta_3 - \theta_4\right) \tag{36}$$

$$D_3 = \left(\gamma_3 - 1\right)\sin\left(\theta_3 - \theta_4\right)\cos\left(\theta_3 - \theta_2\right) \tag{37}$$

$$D_4 = \left(\gamma_4 - \gamma_3\right)\sin\left(\theta_3 - \theta_2\right)\cos\left(\theta_3 - \theta_4\right) \tag{38}$$

In order to model the whole system, the dynamic of the actuator must be included into the dynamics of the FBM-SDF (26). A schematic diagram of the DC motor is represented in Fig. 2, where $L$ and $R$ represent the inductance and the armature resistance, $i(t)$ and $V_{in}(t)$ are the current and voltage input, respectively. $J$ and $B$ is the inertia moment and the friction coefficient of the output shaft. $T_L$, $T_m$ and $V_b$ is the load torque, the magnetic motor torque and the Back electromotive force of the motor, respectively. The motor constant is represented by $K_f$ and the constant of the back electromotive force is represented by $K_b$.

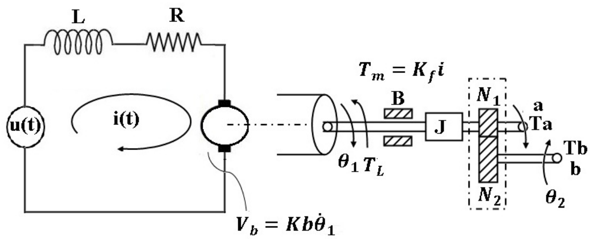



**Fig. 2.** Schematic diagram of a DC Motor.

The dynamic model of the DC motor [1], take into account the electrical and mechanical subsystems. Using Kirchhoff's second law, the closed loop circuit of Fig. 2 can be written as (39).

$$L\frac{di\left(t\right)}{dt} + Ri\left(t\right) = V_{in}\left(t\right) - K_b\dot{\theta}_1 \tag{39}$$

By using the Newton's second law in the mechanical part of the DC motor, the equation (40) is obtanied, where $T_a$ and $T_b$ is the output torque of the shaft $a$ and $b$, respectively (see Fig. 2).

$$T_m - B\dot{\theta}_1 - T_a - T_L = J\ddot{\theta}_1 \tag{40}$$

The mechanical transmission among the two gears in the shafts is expressed in (41), where $r_i$ and $N_i$ $\forall\ i = 1, 2$ is the radius and the number of teeth of the gears.

$$\frac{T_b}{T_a} = \frac{\dot{\theta}_1}{\dot{\theta}_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1} = n \tag{41}$$

Substituting $T_a$ from (40) to (41), the torque applied to the mechanical system is written as (42).

$$T_b = n\left(T_m - T_L - B\dot{\theta}_1 - J\ddot{\theta}_1\right) \tag{42}$$

Using the relation $\dot{\theta}_1 = n\dot{\theta}_2$ in (41), $T_m = K_f i$ and $T_L = 0$, the dynamic equation of the DC motor is given by (43)-(44).

$$T_b = nK_f i\,(t) - n^2 B\dot{\theta}_2 - n^2 J\ddot{\theta}_2 \tag{43}$$

$$L\frac{di\,(t)}{dt} + Ri\,(t) = V_{in}\,(t) - nk_b\dot{\theta}_2 \tag{44}$$

Hence, the coupled dynamics of the DC motor with the FBM-SDF is given by including the motor's dynamics (43), (44) into (26). Let the state variable vector $x = [x_1, x_2, x_3]^T = [\theta_2, \dot{\theta}_2, i]^T$ and the input vector $u = V_{in}$, the coupled dynamics in a state space representation of the DC motor with the FBM-SDF is given by (45).

$$\begin{aligned} \dot{x} &= f(\vec{x}, u(t), t) \\ &= \begin{bmatrix} x_2 \\ A_0\left[A_1 x_2^2 + A_2 x_2 + nK_f x_3 + A_3\right] \\ \frac{1}{L}\left(u\,(t) - nK_b x_2 - Rx_3\right) \end{bmatrix} \end{aligned} \tag{45}$$

where:

$$A_0 = \frac{1}{A\,(x_1) + n^2 J1} \tag{46}$$

$$A_1 = -\frac{1}{2}\frac{dA\,(x_1)}{dx_1} \tag{47}$$

$$A_2 = -\left(C\gamma_2^4 + n^2 B\right) \tag{48}$$

$$A_3 = -k\gamma_4\,(\theta_4 - \theta_{4,0}) \tag{49}$$

## 3. OPTIMAL STRATEGY

The goal of the dynamic approach statement of the optimum synthesis of the FBM-SDF is not only take into account the kinematics of the mechanism. The dynamic approach exposed, is developed in the framework of the mechatronic design approach, where structural and control aspects should be integrated in the design of systems. This work represents a first step to achieve the results mentioned in this approach.

### 3.1. Design variable vector

Once the dynamic and kinematic analysis of the FBM-SDF were carried out, it should be clear that the vector of design variables is composed by the dimensions of the links and the angle $\theta_1$ of the reference link. Those variables are grouped into the vector $\vec{p} \in R^5$ (50).

$$\vec{p} = (p_1, p_2, p_3, p_4, p_5)^T = (L_1, L_2, L_3, L_4, \theta_1)^T \tag{50}$$

### 3.2. Design performance

The motion amplitude of the rocker is showed in fig. 3. The maximum value of this motion is considered as the performance function to be maximize. The performance function is expressed in (51), where $\theta_{4max}$ and $\theta_{4min}$ are computed according to the four-bar configuration as in (52)-(53).

$$\Phi_1 = \theta_{4max} - \theta_{4min} \tag{51}$$

$$\theta_{4max} = \pi + |p_5| sgn(p_5) - cos^{-1}\left(\frac{p_1^2 + p_4^2 - (p_3 - p_1)^2}{2p_1p_4}\right) \tag{52}$$

$$\theta_{4min} = \pi + |p_5| sgn(p_5) - cos^{-1}\left(\frac{p_1^2 + p_4^2 - (p_3 + p_1)^2}{2p_1p_4}\right) \tag{53}$$

### 3.3. Design constraints

A set of design constraints are established to obtain a set of values which produce a suitable FBM-SDF.

#### 3.3.1. Grashof's law

In order to fulfill a crank-rocker four-bar linkage, the mechanical elements of the FBM-SDF must fulfill the Grashof's law. Hence, the sum of the shortest and largest link lengths can not be greater than the sum of the two remaining link lengths. Denoting $\bar{s}$ and $\bar{l}$ the shortest and the largest links of the four-bar mechanism and $\bar{p}$ and $\bar{q}$ the remaining link lengths, Grashof's law is established as detailed in (54).

$$\bar{s} + \bar{l} \leq \bar{p} + \bar{q} \tag{54}$$

In the problem tackled in this work, Grashof's law is given by (55).

$$p_2 + p_3 \leq p_1 + p_4 \tag{55}$$

In addition, to ensure that the solution method produces Grashof mechanisms, it must fulfill the conditions established in (56) and (57).

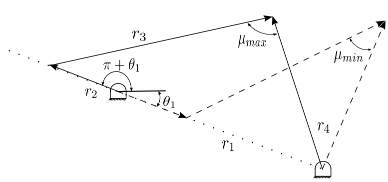



**Fig. 3.** Motion amplitude $\Phi_1$ of the FBM-SDF.

$$p_1 \leq p_3 \tag{56}$$

$$p_4 \leq p_3 \tag{57}$$

3.3.2. Transmission angle

One of the most characteristics used in order to evaluate the quality of a linked mechanism is the transmission angle. The transmission angles $\mu$ for the FBM-SDF is defined as [6]: *the angle between the output link $L_4$ and the coupler link $L_3$. Generally taken as the absolute value of the pair of acute angle corners formed at the intersection of the two links and varies continuously from a maximum value to a minimal, as the linkage passes through its range of motion.* Hence the transmission angle of the FBM-SDF is defined in (58).

$$\mu = \left| \begin{array}{cc} \pi - |\theta_3 - \theta_4| & if\, |\theta_3 - \theta_4| > \frac{\pi}{2} \\ |\theta_3 - \theta_4| & if\, |\theta_3 - \theta_4| \leq \frac{\pi}{2} \end{array} \right. \tag{58}$$

A recommended transmission angle is that it must be greater than 45° along the crank cycle. So, the transmission angle must fulfill the equation (59).

$$\mu \geq 45° \tag{59}$$

3.3.3. Motion symmetry

A necessary characteristic of the rocker motion is a symmetrical motion around vertical axis. Hence, in order to guarantee the symmetrical motion, the equality constraint (60) is established. In Fig. 3 the symmetrical motion constraint is shown.

$$180° - \theta_{4max} = \theta_{4min} \tag{60}$$

3.3.4. System size

Due to the available space, the mechanical elements of the FBM-SDF must fulfil dimensional constraints. In order to do this, the length of each link is determined between $0.05m$ and $0.5m$, so that the constraints are presented in (61) - (64).

$$0.05 \leq p_1 \leq 0.5 \tag{61}$$

$$0.05 \leq p_2 \leq 0.5 \tag{62}$$

$$0.05 \leq p_3 \leq 0.5 \tag{63}$$

$$0.05 \leq p_4 \leq 0.5 \tag{64}$$

On the other hand, the angle between the horizontal axis and the reference bar ($L_1$) is limited between 45° and −45°, as pointed out in (65).

$$-45° \leq p_5 \leq 45° \tag{65}$$

## 4. DYNAMIC APPROACH STATEMENT OF DIMENSIONAL SYNTHESIS

The dynamic approach statement for the dimensional synthesis of the FBM-SDF is formulated as a mono-objective dynamic optimization problem (MODOP). The MODOP consists on finding the optimal design variables $\vec{p}^* \in R^5$ which maximize the performance function (66) subject to inherent inequality and equality constraints in the design (68) - (72), the dynamic behavior of the FBM-SDF represented in state variables (67). It is important to remark that the inequality constraint (71) is a dynamic constraint which is evaluated using the profile of the state vector $x$.

$$\begin{array}{cc} Max & \Phi_1(\vec{p}) = (\theta_{4max} - \theta_{4min})^2 \\ \vec{p} \in \mathbb{R}^5 & \end{array} \tag{66}$$

subject to:

$$\dot{x} = f(x, \vec{p}, t) \tag{67}$$

$$g_1(\vec{p}) = p_2 + p_3 - p_1 - p_4 \leq 0 \tag{68}$$

$$g_2(\vec{p}) = p_1 - p_3 \leq 0 \tag{69}$$

$$g_3(\vec{p}) = p_4 - p_3 \leq 0 \tag{70}$$

$$g_4(\vec{p}, t) = \frac{\pi}{4} - \mu(p, t) \leq 0 \tag{71}$$

$$h_1(\vec{p}) = \pi - \theta_{4max} - \theta_{4min} = 0 \tag{72}$$

$$\begin{array}{l} 0.05 \leq p_i \leq 0.5 \quad i = 1, \ldots, 4 \\ -\frac{\pi}{4} \leq p_5 \leq \frac{\pi}{4} \end{array} \tag{73}$$

## 5. SWARM INTELLIGENCE STRATEGY

Currently, Swarm Intelligence Algorithms (SIA's) are a suitable option in order to solve optimization problems. One of the most popular algorithm is the Artificial Bee Colony (ABC), which is an algorithm based on the foraging behaviour of the honey bee [14]. Originally, this algorithm deals with unconstrained nonlinear optimization problems. However, due to the engineering problems usually include a set of constraints, a Modified Artificial Bee Colony (M-ABC) for constrained numerical optimization version [4] was used in this work. In this section, a brief explanation of the main aspects of the ABC algorithm are presented. After that, the M-ABC algorithm is presented, remarking the computational implementation that was carried out in the present work.

### 5.1. Artificial Bee Colony

In [14], the process of the search of nectar in the flowers by the honey bees has been seen as an optimization process. The way that this kind of social insects manage to focus efforts on areas with high amounts of food sources has been modelled as a heuristic for optimization. Two behaviours are used in order to do this: the recruitment of bees into a food source and the abandonment of a source. It is important to remark that in the ABC algorithm, the solutions of the problem are represented by the food sources, not by the bees. The bees act as variation operators, because when one of them comes to a food source, calculates a new candidate solution based on the first one.

In the ABC algorithm, the colony of artificial bees consists of three kinds of bees: employed, onlooker and scout bees. Usually, the number of employed bees is equal to the number of food sources and each employed bee will be assigned to each one of the sources. When the bee get to the food source, it will calculate a new solution (the bee will fly to other nearby food source) from it and retain the best solution based on a greedy selection. The number of onlooker bees is usually the same as the number of employed bees. This kind of bees are assigned to a food source according to the profitability of such source. In the same way that the employed bees, the onlooker bees will calculate a new solution based on their assigned food source. When a food source does not improve after a certain number of iteration, the food source is abandoned and is replaced by a food source randomly assigned. The user-defined parameters required by the ABC algorithm are: the number of food sources or solutions $SN$, the total number of iterations or cycles $MCN$ and the number of cycles that a non improved food source will be kept before being replaced by a new food source *limit*. It is important to remark that an advantage of the ABC algorithm is that the solutions are real-encoding. Therefore, it is easily used in engineering design problems.

### 5.2. Modified Artificial Bee Colony

The M-ABC algortihm is shown in Fig. 4. In this algorithm, the variation operator used by both, employed and onlooker bees in order to generate a new candidate solution $\nu_i^g$ includes a recombination mechanism. The variation operator is given by:

$$\nu_i^g = \begin{cases} x_{i,j}^g + \phi_j \cdot (x_{i,j}^g - x_{k,j}^g) & \text{, if rand(0,1) < MR} \\ x_{i,j}^g & \text{, otherwise} \end{cases} \tag{74}$$

where the user-defined recombination mechanism is established as $0 \leq MR \leq 1$. On the other hand, in order to select the best food source, a tournament selection is carried out based on the set of three rules defined in [2]. Such set of rules is given as follows:

- Between two feasibles food sources, the one with the best objective function value is preferred.
- Between a feasible food source and an infeasible food source, the feasible one is preferred.
- Between two infeasible food sources, the one with the lowest value of the sum of constraint violation is preferred.

In the M-ABC algorithm, a dynamic tolerance for equality constraints is proposed. Such mechanism is established as follows:

$$\epsilon(g+1) = \frac{\epsilon(g)}{dec} \tag{75}$$

where $g$ is the current iteration and $dec$ is the decreasing rate value of each iteration ($dec > 1$). The aims is to start with a larger feasible region than the original region.

The above, in order to more easily meet the equality constraints at the beginning of the iterations. An easy way to compute the decreasing rate value is given by:

$$dec = e^{\left(\frac{ln(\epsilon_0) - ln(\epsilon_f)}{MCN}\right)} \tag{76}$$

where $\epsilon_0$ is the initial tolerance value and $\epsilon_f$ is the final tolerance value.

In the M-ABC algorithm, a smart flight operator is included. This operator combines three elements: (1) the information of the solution to be replaced $x_i^g$, the solution is used as a reference point in order to generate a new solution, after this the solution is eliminated, (2) the location of this new solution will be biased by the best solution $x_B^g$, the aim of include this food source is to find a feasible solution or, at least, an infeasible solution closer to the feasible region, and (3) a solution which is randomly selected $x_k^g$ in order to avoid a full attraction by the best solution so far. The new solution taking into account the smart flight operator is computed as follows:

$$\nu_{i,j}^g = x_{i,j}^g + \phi \cdot (x_{k,j}^g - x_{i,j}^g) + (1 - \phi) \cdot (x_{B,j}^g - x_{i,j}^g) \tag{77}$$

Finally, the boundary constraint-handling used for the design variables is implemented as follows:

$$\nu_{i,j}^g = \begin{cases} 2 * L_j - \nu_{i,j}^g & \text{, if } \nu_{i,j}^g < L_j \\ 2 * U_j - \nu_{i,j}^g & \text{, if } \nu_{i,j}^g > U_j \\ \nu_{i,j}^g & \text{, otherwise} \end{cases} \tag{78}$$

where $\nu_{i,j}^g$ is the $j-th$ variable of the $i-th$ candidate solution at iteration $g$, $L_j$ is the lower limit and $U_j$ is the upper limit of the $j-th$ variable.

### 5.3. Implementation issues

As mentioned in this section, a M-ABC algorithm was used. However, due to the type of optimization problem some adaptations were carried out:

- Due to the solution of the optimization problem must meets the Grashof's criteria, the set of three rules was not applied directly, in a step prior to the tournament was verified that the food sources participants meet the Grashof's criteria. In fact, in the lines 15, 21 and 24 of the M-ABC algorithm, the selection between the contenders is based on the next rules:

    1. When both food sources meet the Grashof's criteria, the set of three rules above defined is applied.
    2. When a food source meets the Grashof's criteria and the other one does not meet such criteria, the Grashof's one is preferred.
    3. When both food sources do not meet the Grashof's criteria, the one with the lowest value of the sum of constraint violation is preferred.

```
1   BEGIN
2     Initialize the set of food sources $x_i^0$, $i = 1, \ldots, SN$
3     Evaluate each $x_i^0$, $i = 1, \ldots, SN$
4     $g = 1$
5     IF There are equality constraints
6       Initialize $\epsilon(g)$
7     END IF
8     REPEAT
9       IF There are equality constraints
10        Evaluate each $x_i^0$, $i = 1, \ldots, SN$ with $\epsilon(g)$
11      END IF
12      FOR $i = 1$ TO $SN$
13        Generate $\nu_i^g$ with $x_i^{g-1}$ by using Eq. 74
14        Evaluate $\nu_i^g$
15        IF $\nu_i^g$ is better than $x_i^{g-1}$(based on feasibility criteria in Section 5.2)
16          $x_i^g = \nu_i^g$
17        ELSE
18          $x_i^g = x_i^{g-1}$
19      END FOR
20      FOR $i = 1$ TO $SN$
21        Select food source $x_i^g$ based on binary tournament selection (Section 5.2)
22        Generate $\nu_i^g$ with $x_i^g$ by using Eq. 74
23        Evaluate $\nu_i^g$
24        IF $\nu_i^g$ is better than $x_i^g$(based on feasibility criteria in Section 5.2)
25          $x_i^g = \nu_i^g$
26        END IF
27      END FOR
28      Apply the smart flight by the scout bees (Eq. 77) for those
        solutions whose limit to be improved has been reached
29      Keep the best solution so far
30      $g = g + 1$
31      IF There are equality constraints
32        Update $\epsilon(g)$ by using Eq. FF
33      END IF
34    UNTIL $g = MCN$
35  END
```

**Fig. 4.** Modified Artificial Bee Colony Algortihm (M-ABC).

- In order to apply the smart flight operator (line 28 of the M-ABC algorithm), the best source is needed, therefore, the search of such source is carried out as follows:

  1. In the current distribution of food sources, a search by a food source that meets the Grashof's law is carried out. Once that a food source is found, the search continues for this type of food sources in order to compare and select (based on the three rules above mentioned) the best of the whole distribution.
  2. In case that in the current distribution of food sources there are not Grashof's

food sources, a search for the food source with the lowest value of the sum of constraint violation is carried out.

- Due to the inequality constraint $g_4$ of the optimization problem is a dynamic constraint, it is evaluated only when the food source meets the Grashof's criteria.

It is important to remark that the modifications to the M-ABC algorithm are due to the type of optimization problem that is solved in the present work. However, the original approach in order to solve an optimization problem was preserved.

## 6. RESULTS AND DISCUSSION

In the present work, a set of 10 independent runs was carried out. A fixed set of values for the M-ABC parameters was used in all runs as follows: number of solutions $SN = 20$, maximum cycle number $MCN = 10000$, $Limit = MCN/(2 * SN) = 250$ and the modification rate $MR = 0.8$. On the other hand, in order to evaluate the equality constraint, the initial and final desired values of $\epsilon_0$ and $\epsilon_f$ were 1.0 and 0.01, respectively. The M-ABC algorithm was coded in Matlab® R2008a and was run in a Laptop computer with 6 GB RAM, Intel® Core i5 processor at 2.5 GHz, and Microsoft Windows® 7 OS.

The results of computational experiments are shown in Table 1. Also, the statistical results by the independent runs can be observed in Table 2. Finally, the time required per run is shown in Table 3.

| Run | Vector of design variables | Objective function |
|---|---|---|
| 1 | 0.434183161 0.111987151 0.434193070 0.200010277 -0.198555316 | 1.474881068 |
| 2 | 0.436286674 0.112199817 0.436288484 0.200024842 -0.197834785 | 1.480289545 |
| 3 | 0.462774558 0.113985089 0.462909518 0.200010509 -0.183396807 | 1.527252312 |
| 4 | 0.499960734 0.116714229 0.499963500 0.200009037 -0.169495104 | 1.602442026 |
| 5 | 0.499314875 0.116673002 0.499315293 0.200004459 -0.169621422 | 1.601355458 |
| 6 | 0.496355846 0.116497626 0.496356328 0.200014750 -0.170800563 | 1.596298246 |
| 7 | 0.499756750 0.116698292 0.499759307 0.200000141 -0.169562651 | 1.602141700 |
| 8 | 0.499882643 0.116710523 0.499883948 0.200010631 -0.169479315 | 1.602308234 |
| 9 | 0.496954796 0.116528730 0.496956381 0.200001937 -0.170573303 | 1.597376571 |
| 10 | 0.496757589 0.116506497 0.496765845 0.200040121 -0.170134742 | 1.596069930 |

**Tab. 1.** Details of the solutions obtained by the M-ABC algorithm.

| | |
|---|---|
| Best | **1.602442026** |
| Mean | **1.568041509** |
| Worst | **1.474881068** |
| Std. Dev. | **0.050117723** |

**Tab. 2.** Statistical results for the independent runs by the M-ABC algorithm.

From Table 1 and Table 2 it can be observed that the behaviour of the M-ABC algorithm is stable computationally speaking. The above because the best and worst solutions shown a similar performance as it is indicated in the standard deviation. On

| **Run** | **Time required/Hrs** |
|---|---|
| 1 | **1.10** |
| 2 | **1.13** |
| 3 | **1.06** |
| 4 | **0.94** |
| 5 | **0.94** |
| 6 | **0.83** |
| 7 | **0.82** |
| 8 | **0.86** |
| 9 | **0.96** |
| 10 | **0.81** |
| **Average** | **0.94** |

**Tab. 3.** Time required at each independent run by the M-ABC algorithm.

the other hand, the computational time is not expensive in order to obtain a solution of the optimization problem, based on the results from Table 3.

### 6.1. Mechanical analysis of solutions

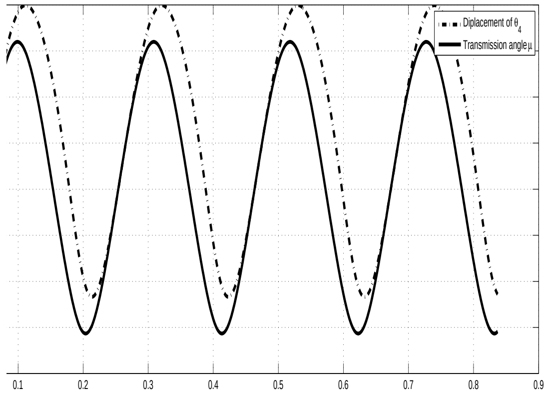
It is worth recalling that the goal of the optimization problem is to obtain a set of values for the FBM-SDF mechanism. Therefore, the best and worst solutions obtained by the M-ABC algorithm were subject to simulation in order to obtain more information about the mechanical performance of the resulting mechanism. The simulation results for the displacement angle ($\theta_4$) of the rocker and the transmission angle ($\mu$) for the best and worst solutions based on the solutions in Table 1, are shown in Fig. 5.

As it mentioned in section 3, the optimal set of values by the FBM mechanism, should allow a symmetric displacement of the rocker around the vertical axis ($\frac{\pi}{2}$ *rad* is the reference value), also in order to obtain a high quality of the whole mechanism, the transmission angle should be greater than $\frac{\pi}{4}$ *rad* and closer to $\frac{\pi}{2}$ *rad*. As it can be observed in Fig. 5, both solutions are inside the requirements established on the optimal strategy.

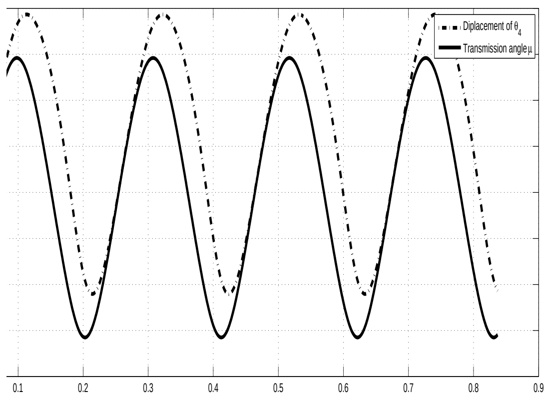
## 7. CONCLUSIONS

In the present work a dynamic approach in order to obtain the optimum synthesis of a FBM mechanism using a swarm intelligence algorithm was presented. In order to do this, the mechanical synthesis is established as a dynamic optimization problem, where the dynamic model of the system is taking into account as well as a set of constraints and an objective function. It is important to remark that one of the constraints is a dynamic constraint.

A swarm intelligence algorithm called Modified-Artificial Bee Colony was implemented in order to obtain the solution of the optimization problem. A set of independent




*a*)




*b*)

**Fig. 5.** Simulation results of the angular displacement of the rocker $\theta_4$ and transmission angle $\mu$. Best solution *a*) and worst solution *b*) of Table 1.

runs were carried out in order to test the performance and behaviour of the algorithm and the solutions, respectively.

The results presented in this work, can be used to show that the dynamic approach proposed is suitable for this type of planar mechanism. Also, the computational implementation of the M-ABC algorithm allows its applications without great changes or adaptations in order to solve real-world problems.

Future work will include design the mechanism in the mechatronic framework, where a concurrent design methodology is implemented. That concurrent design methodology deals with the optimal design at the same time of the mechanical structure and the controller.

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